# Autumn Scheme of Learning

# Year 6

#MathsEveryoneCan

2019-20

White Rose Maths

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# Welcome

Welcome to the White Rose Maths' new, more detailed schemes of learning for 2019-20.

We have listened to all the feedback over the last 2 years and as a result of this, we have made some changes to our primary schemes. *They are bigger, bolder and more detailed than before.*

The new schemes still have the *same look and feel* as the old ones, but we have tried to provide more detailed guidance. We have worked with enthusiastic and passionate teachers from up and down the country, who are experts in their particular year group, to bring you additional guidance. *These schemes have been written for teachers, by teachers.*

*We all believe that every child can succeed in mathematics.* Thank you to everyone who has contributed to the work of White Rose Maths. It is only with your help that we can make a difference.

We hope that you find the schemes of learning helpful. As always, get in touch if you or your school want support with any aspect of teaching maths.

If you have any feedback on any part of our work, do not hesitate to contact us. Follow us on Twitter and Facebook to keep up-to-date with all our latest announcements.

**Thanks from the White Rose Maths Team**

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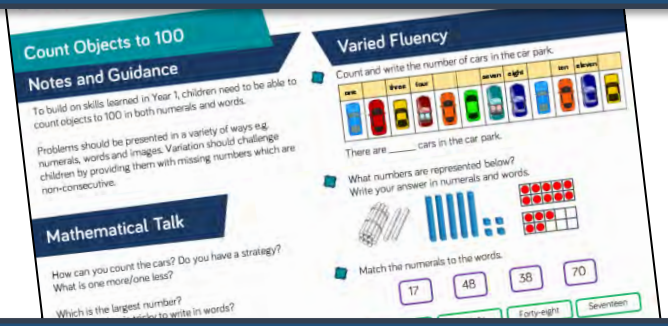
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# What's included?

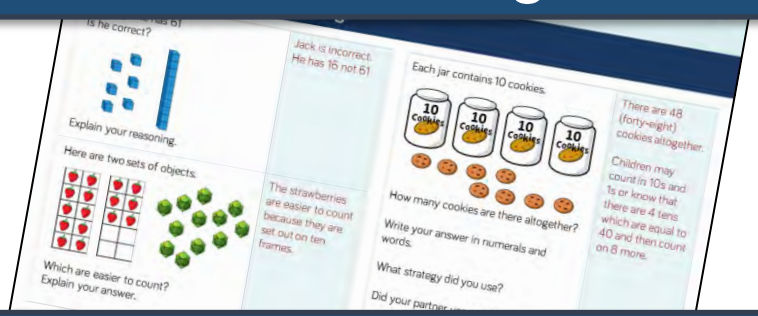
Our schemes include:

- Small steps progression. These show our blocks broken down into smaller steps.
- Small steps guidance. For each small step we provide some brief guidance to help teachers understand the key discussion and teaching points. This guidance has been written for teachers, by teachers.
- A more integrated approach to fluency, reasoning and problem solving.
- Answers to all the problems in our new scheme.
- This year there will also be updated assessments.
- We are also working with Diagnostic Questions to provide questions for every single objective of the National Curriculum.

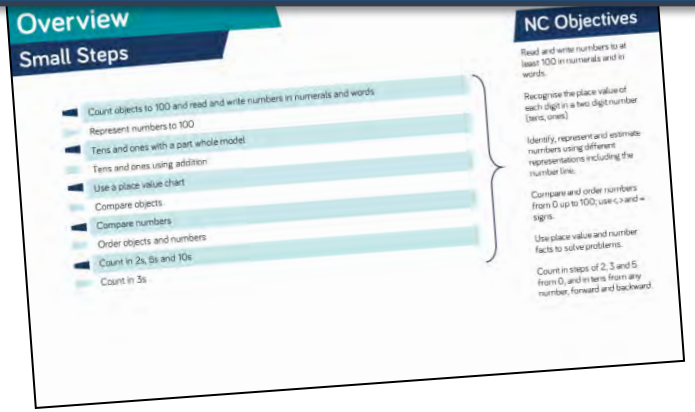
## Teaching notes and examples




## Answers to Reasoning Questions




## Small Steps Guidance






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# How to use the small steps

We were regularly asked how it is possible to spend so long on particular blocks of content and National Curriculum objectives.

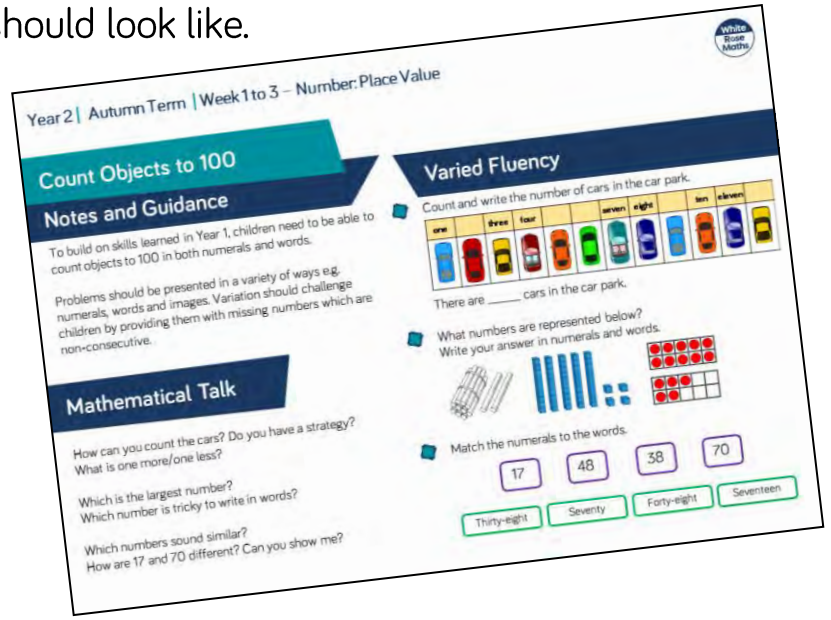
We know that breaking the curriculum down into small manageable steps should help children understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. In our opinion, it is better to follow a small steps approach.

As a result, for each block of content we have provided a "Small Step" breakdown. We recommend that the steps are taught separately and would encourage teachers to spend more time on particular steps if they feel it is necessary. Flexibility has been built into the scheme to allow this to happen.

# Teaching notes

Alongside the small steps breakdown, we have provided teachers with some brief notes and guidance to help enhance their teaching of the topic. The "Mathematical Talk" section provides questions to encourage mathematical thinking and reasoning, to dig deeper into concepts.

We have also continued to provide guidance on what varied fluency, reasoning and problem solving should look like.






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# Teaching for Mastery

These overviews are designed to support a mastery approach to teaching and learning and have been designed to support the aims and objectives of the new National Curriculum.

The overviews:

- have number at their heart. A large proportion of time is spent reinforcing number to build competency
- ensure teachers stay in the required key stage and support the ideal of depth before breadth.
- ensure students have the opportunity to stay together as they work through the schemes as a whole group
- provide plenty of opportunities to build reasoning and problem solving elements into the curriculum.

For more guidance on teaching for mastery, visit the NCETM website:

https://www.ncetm.org.uk/resources/47230

# Concrete - Pictorial - Abstract

We believe that all children, when introduced to a new concept, should have the opportunity to build competency by taking this approach.

Concrete – children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – alongside this children should use pictorial representations. These representations can then be used to help reason and solve problems.

Abstract – both concrete and pictorial representations should support children's understanding of abstract methods.

Need some CPD to develop this approach? Visit www.whiterosemaths.com for find a course right for you.



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# Supporting resources

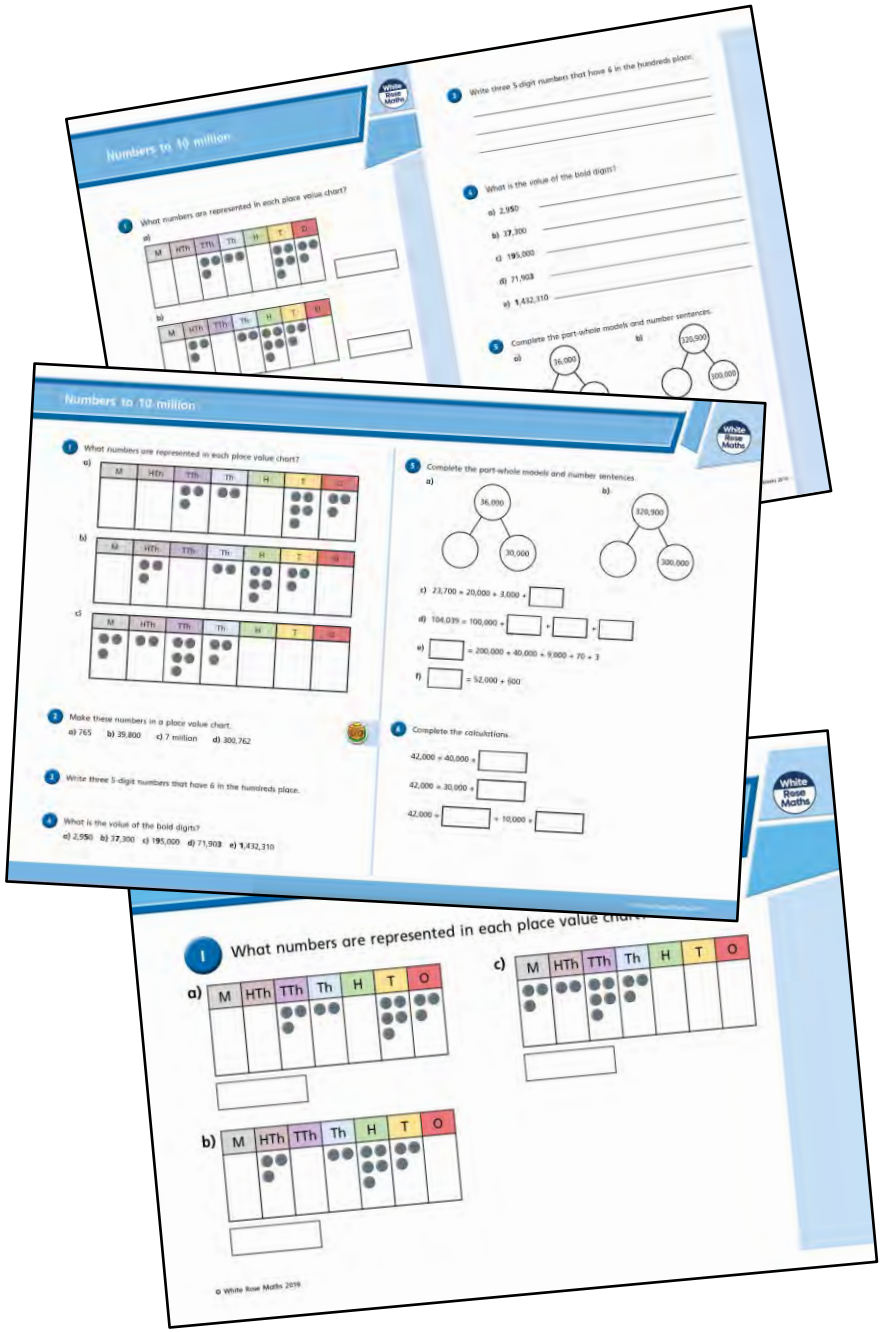
NEW for 2019-20!

We have produced supporting resources for every small step from Year 1 to Year 8.

The worksheets are provided in three different formats:

- Write on worksheet – ideal for children to use the ready made models, images and stem sentences.
- Display version – great for schools who want to cut down on photocopying.
- PowerPoint version – one question per slide. Perfect for whole class teaching or mixing questions to make your own bespoke lesson.

For more information visit our online training and resources centre www.resources.whiterosemaths.com or email us directly at support@whiterosemaths.com






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# Training

White Rose Maths offer a plethora of training courses to help you embed teaching for mastery at your school.

Our popular JIGSAW package consists of five key elements:

- CPA
- Bar Modelling
- Mathematical Talk & Questioning
- Reasoning & Problem Solving
- Thinking through Variation

For more information and to book visit our website [www.whiterosemaths.com](http://www.whiterosemaths.com)

NEW for 2019-20!

We have made the above courses available in a digital format. You can now have CPD whenever you want, wherever you want in easy to digest bite size chunks.

Find out more at [www.resources.whiterosemaths.com](http://www.resources.whiterosemaths.com)









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# FAQs

### *If we spend so much time on number work, how can we cover the rest of the curriculum?*

Children who have an excellent grasp of number make better mathematicians. Spending longer on mastering key topics will build a child's confidence and help secure understanding. This should mean that less time will need to be spent on other topics.

In addition, schools that have been using these schemes already have used other subjects and topic time to teach and consolidate other areas of the mathematics curriculum.

### *Should I teach one small step per lesson?*

Each small step should be seen as a separate concept that needs teaching. You may find that you need to spend more time on particular concepts. Flexibility has been built into the curriculum model to allow this to happen. This may involve spending more than one lesson on a small step, depending on your class' understanding.

### *How do I use the fluency, reasoning and problem solving questions?*

The questions are designed to be used by the teacher to help them understand the key teaching points that need to be covered. They should be used as inspiration and ideas to help teachers plan carefully structured lessons.

### *How do I reinforce what children already know if I don't teach a concept again?*

The scheme has been designed to give sufficient time for teachers to explore concepts in depth, however we also interleave prior content in new concepts. E.g. when children look at measurement we recommend that there are lots of questions that practice the four operations and fractions. This helps children make links between topics and understand them more deeply. We also recommend that schools look to reinforce number fluency through mental and oral starters or in additional maths time during the day.



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# Meet the Characters

Children love to learn with characters and our team within the scheme will be sure to get them talking and reasoning about mathematical concepts and ideas. Who's your favourite?



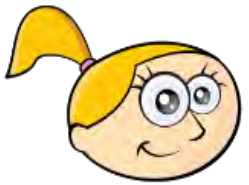
Teddy



Rosie



Mo



Eva



Alex



Jack



Whitney



Amir



Dora



Tommy



Dexter



Ron



Annie



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| | Week 1 | Week 2 | Week 3 | Week 4 | Week 5 | Week 6 | Week 7 | Week 8 | Week 9 | Week 10 | Week 11 | Week 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Autumn | Number: Place Value | | Number: Addition, Subtraction, Multiplication and Division | | | | Number: Fractions | | | | Geometry: Position and Direction | Consolidation |
| Spring | Number: Decimals | | Number: Percentages | | Number: Algebra | | Measurement: Converting Units | Measurement: Perimeter, Area and Volume | | Number: Ratio | | Consolidation |
| Summer | Geometry: Properties of Shape | | Problem Solving | | | Statistics | | Investigations | | | | Consolidation |



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Autumn - Block 1

# Place Value

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# Overview

## Small Steps

- Numbers to ten million
- Compare and order any number
- Round any number
- Negative numbers

## NC Objectives

Read, write, order and compare numbers up to 10,000,000 and determine the value of each digit.

Round any whole number to a required degree of accuracy.

Use negative numbers in context, and calculate intervals across zero.

Solve number and practical problems that involve all of the above.



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# Numbers to Ten Million

## Notes and Guidance

Children need to read, write and represent numbers to ten million in different ways.
Numbers do not always have to be in the millions – they should see a mixture of smaller and larger numbers, with up to seven digits. The repeating patterns of ones, tens, hundreds, ones of thousands, tens of thousands, hundreds of thousands could be discussed and linked to the placement of commas or other separators.

## Mathematical Talk

Why is the zero in a number important when representing large numbers?

What strategies can you use to match the representation to the correct number?

How many ways can you complete the partitioned number?

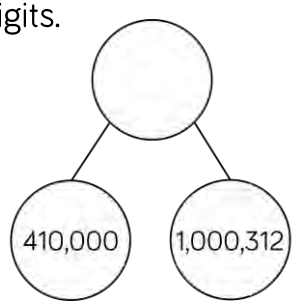
What strategy can you use to work out Teddy's new number?

## Varied Fluency

Match the representations to the numbers in digits.

One million, four hundred and one thousand, three hundred and twelve.

| M | HTh | TTh | Th | H | T | O |
|---|---|---|---|---|---|---|
| ● | | ● ● | ● | ● ● | ● | ● ● |




1,401,312 | 1,041,312 | 1,410,312

Complete the missing numbers.

6,305,400 = _____ + 300,000 + _____ + 400

7,001,001 = 7,000,000 + _____ + _____

42,550 = _____ + _____ + _____ + 50

Teddy's number is 306,042
He adds 5,000 to his number.
What is his new number?



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# Numbers to Ten Million

## Reasoning and Problem Solving

Put a digit in the missing spaces to make the statement correct.

$4{,}62_\,{,}645 < 4{,}623{,}64_$

Is there more than one option? Can you find them all?

The first digit can be 0, 1, 2 or 3
When the first digit is 0, 1 or 2, the second digit can be any.
When the first digit is 3, the second digit can be 6 or above.

Dora has the number 824,650

She subtracts forty thousand from her number.

She thinks her new number is 820,650

Is she correct?

Explain how you know.

Dora is incorrect because she has subtracted 4,000 not 40,000
Her answer should be 784,650

Use the digit cards and statements to work out my number.




- The ten thousands and hundreds have the same digit.
- The hundred thousand digit is double the tens digit.
- It is a six-digit number.
- It is less than six hundred and fifty-five thousand.

Is this the only possible solution?

Possible solutions:

653,530
653,537
650,537
650,533



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# Compare and Order

## Notes and Guidance

Children will compare and order whole numbers up to ten million using numbers presented in different ways.

They should use the correct mathematical vocabulary (greater than/less than) alongside inequality symbols.

## Mathematical Talk

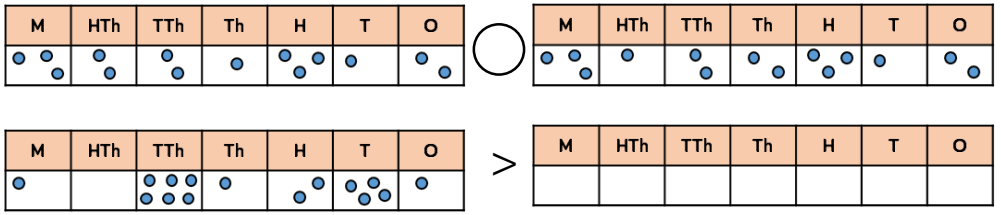
What is the value of each digit in the number?
What is the value of _____ in this number?

What is the value of the whole? Can you suggest other parts that make the whole?

What do you know about the covered number?
What could the number be? What must the number be? What can't the number be?

## Varied Fluency

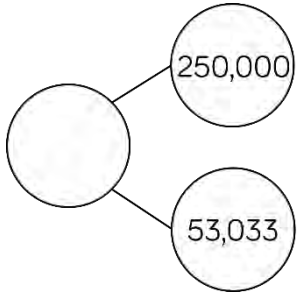
Complete the statements to make them true.




What number could the splat be covering?

Three hundred and thirteen thousand and thirty-three






Greatest → Smallest

A house costs £250,000
A motorised home costs £100,000
A bungalow is priced halfway between the two.
Work out the price of the bungalow.



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# Compare and Order

## Reasoning and Problem Solving

Eva has ordered eight 6-digit numbers.

The smallest number is 345,900

The greatest number is 347,000

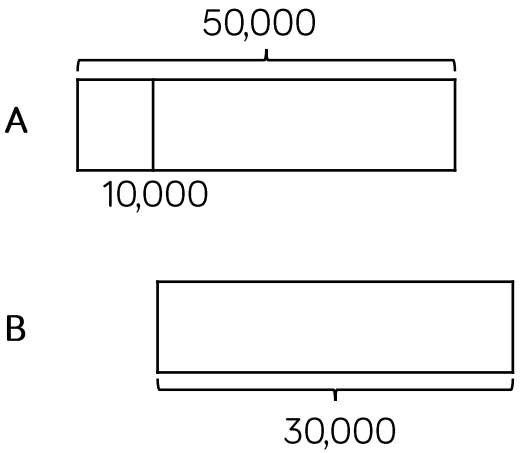
All the other numbers have a digit total of 20 and have no repeating digits.

What are the other six numbers?

Can you place all eight numbers in ascending order?

The other six numbers have to have a digit total of 20 and so must start with 346, _ _ _ because anything between 345,900 and 346,000 has a larger digit total. The final three digits have to add up to 7 so the solution is:
345,900
346,025
346,052
346,205
346,250
346,502
346,520
347,000

Jack draws bar model A.
His teacher asks him to draw another where the total is 30,000




Explain how you know bar B is inaccurate.

Bar B is inaccurate because it starts at 10,000 and finishes after 50,000 therefore it is longer than 40,000



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# Round within Ten Million

## Notes and Guidance

Children build on their prior knowledge of rounding.

They will learn to round any number within ten million.

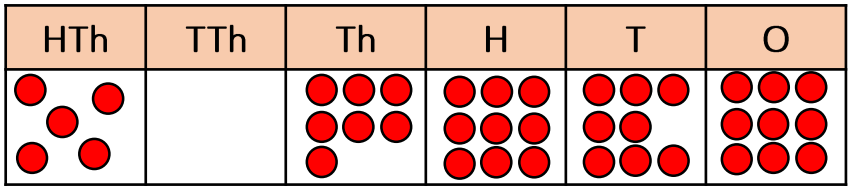
They use their knowledge of multiples and place value columns to work out which two numbers the number they are rounding sits between.

## Mathematical Talk

Why do we round up when the following digit is 5 or above?
Which place value column do we need to look at when we round to the nearest 100,000?
What is the purpose of rounding?
When is it best to round to 1,000? 10,000?
Can you justify your reasoning?

What could/must/can't the missing digit be?
Explain how you know.

## Varied Fluency




Round the number in the place value chart to:

- The nearest 10,000
- The nearest 100,000
- The nearest 1,000,000

Write five numbers that round to the following numbers when rounded to the nearest hundred thousand.

200,000 600,000 1,900,000

Complete the missing digits so that each number rounds to one hundred and thirty thousand when rounded to the nearest ten thousand.

12 __ ,657 1 __ 1,999 13 __ ,001



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# Round within Ten Million

## Reasoning and Problem Solving



My number is 1,350 when rounded to the nearest 10

Mo



My number is 1,400 when rounded to the nearest 100

Rosie

Both numbers are whole numbers.

What is the greatest possible difference between the two numbers?

The greatest possible difference is 104 (1,345 and 1,449)

Whitney rounded 2,215,678 to the nearest million and wrote 2,215,000

Can you explain to Whitney what mistake she has made?

There should be no non-zero digits in the columns after the millions column.

Miss Grogan gives out four number cards.

| 15,987 | 15,813 | 15,101 | 16,101 |
|---|---|---|---|

Four children each have a card and give a clue to what their number is.

Tommy says, "My number rounds to 16,000 to the nearest 1,000"

Alex says, "My number has one hundred."

Jack says, " My number is 15,990 when rounded to the nearest 10"

Dora says, "My number is 15,000 when rounded to the nearest 1,000"

Can you work out which child has which card?

Tommy: 15,813

Alex: 16,101

Jack: 15,987

Dora: 15,101



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# Negative Numbers

## Notes and Guidance

Children continue their work on negative numbers from year 5 by counting forwards and backwards through zero.

They extend their learning by finding intervals across zero.
Number lines, both vertical and horizontal are useful to support this, as these emphasise the position of zero.
Children need to see negative numbers in relevant contexts.

## Mathematical Talk

Are all negative numbers whole numbers?
Why do the numbers on a number line mirror each other from 0?

Why does positive one add negative one equal zero?
Can you use a number line to show this?

Draw me a picture to show 5 subtract 8
Show 5 more than −2 on a number line.
Could Mo really afford the jumper? How do you know?

## Varied Fluency

Use sandcastles (+1) and holes (−1) to calculate.
Here is an example.

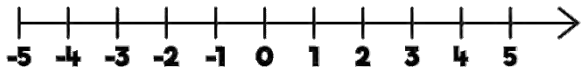
$-2 + 5 =$ 

Two sandcastles will fill two holes. There are three sandcastles left, therefore negative two add five is equal to three.

Use this method to solve:

$3 - 6$ $\quad -7 + 8$ $\quad 5 - 9$

Use the number line to answer the questions.




- What is 6 less than 4?
- What is 5 more than −2?
- What is the difference between 3 and −3?

Mo has £17.50 in his bank account. He pays for a jumper which costs £30
How much does he have in his bank account now?



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# Negative Numbers

## Reasoning and Problem Solving

A company decided to build offices over ground and underground.




Do you agree? Explain why.

No, there would be 41 floors because you need to count floor 0

When counting forwards in tens from any positive one-digit number, the last digit never changes.

When counting backwards in tens from any positive one-digit number, the last digit does change.

Can you find examples to show this?

Explain why this happens.

Possible examples:

9, 19, 29, 39 etc.

9, −1, −11, −21

This happens because when you cross 0, the numbers mirror the positive side of the number line. Therefore, the final digit in the number changes and will make the number bond to 10



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Autumn - Block 2

# Four Operations

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# Overview

## Small Steps

- Add and subtract integers
- Multiply up to a 4-digit number by 2-digit number
- Short division
- Division using factors
- Long division (1)
- Long division (2)
- Long division (3)
- Long division (4)
- Common factors
- Common multiples
- Primes to 100
- Squares and cubes
- Order of operations
- Mental calculations and estimation
- Reason from known facts

## NC Objectives

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Multiply multi-digit numbers up to 4 digits by a 2-digit number using the formal written method of long multiplication.

Divide numbers up to 4 digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context.

Divide numbers up to 4 digits by a 2-digit number using the formal written method of short division, interpreting remainders according to the context.

Perform mental calculations, including with mixed operations and large numbers.

Identify common factors, common multiples and prime numbers.

Use their knowledge of the order of operations to carry out calculations involving the four operations.

Solve problems involving addition, subtraction, multiplication and division.

Use estimation to check answers to calculations and determine in the context of a problem, an appropriate degree of accuracy.



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# Add & Subtract Integers

## Notes and Guidance

Children consolidate their knowledge of column addition and subtraction, reinforcing the language of 'exchange' etc. After showing confidence with smaller numbers, children should progress to multi-digit calculations. Children will consider whether the column method is always appropriate e.g. when adding 999, it is easier to add 1,000 then subtract 1
They use these skills to solve multi-step problems in a range of contexts.

## Mathematical Talk

What happens when there is more than 9 in a place value column?

Can you make an exchange between columns?

How can we find the missing digits? Can we use the inverse?

Is the column method always the best method?

When should we use mental methods?

## Varied Fluency

Calculate.

| | | | | | |
|---|---|---|---|---|---|
| | 3 | 4 | 6 | 2 | 1 |
| + | 2 | 5 | 7 | 3 | 4 |
| | | | | | |

67,832 + 5,258

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 4 | 7 | 6 | 1 | 3 | 2 | 5 |
| – | | 9 | 3 | 8 | 0 | 5 | 2 |
| | | | | | | | |

834,501 – 299,999

A four bedroom house costs £450,000
A three bedroom house costs £201,000 less.
How much does the three bedroom house cost?
What method did you use to find the answer?

Find the missing digits. What do you notice?

| | | | | | | |
|---|---|---|---|---|---|---|
| | 5 | 2 | 2 | 4 | 7 | ? |
| + | 3 | ? | 5 | 9 | 0 | 4 |
| | 9 | 0 | ? | 3 | ? | 2 |

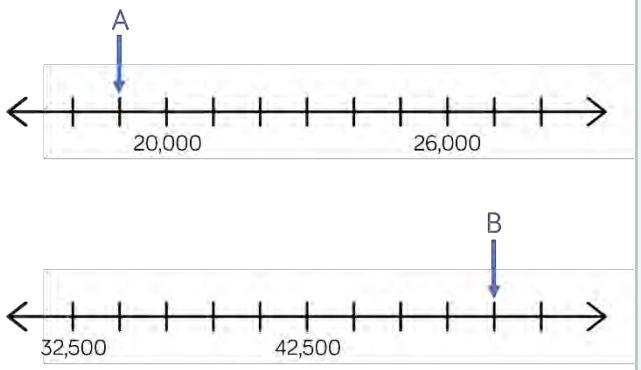


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# Add & Subtract Integers

## Reasoning and Problem Solving

Find the difference between A and B.

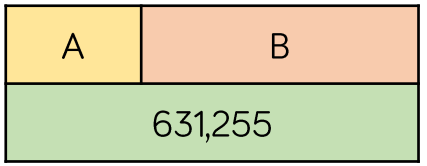



A = 19,000

B = 50,500

The difference is 31,500

Here is a bar model.




A is an odd number which rounds to 100,000 to the nearest ten thousand.
It has a digit total of 30

B is an even number which rounds to 500,000 to the nearest hundred thousand.
It has a digit total of 10

A and B are multiples of 5.

What are possible values of A and B?

Possible answer:

A = 99,255

B = 532,000



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# Multiply 4-digits by 2-digits

## Notes and Guidance

Children consolidate their knowledge of column multiplication, multiplying numbers with up to 4 digits by a 2-digit number. It may be useful to revise multiplication by a single digit first, and then 2- and 3- digit numbers before moving on when ready to the largest calculations.
They use these skills to solve multi-step problems in a range of contexts.

## Mathematical Talk

What is important to remember as we begin multiplying by the tens number?

How would you draw the calculation?

Can the inverse operation be used?

Is there a different strategy that you could use?

## Varied Fluency

- Calculate.

| | | | | |
|---|---|---|---|---|
| | 4 | 2 | 6 | 7 |
| × | | | 3 | 4 |
| | | | | |

| | | | | |
|---|---|---|---|---|
| | 3 | 0 | 4 | 6 |
| × | | | 7 | 3 |
| | | | | |

5,734 × 26

- Jack made cookies for a bake sale.
He made 345 cookies.
The recipe says that he should have 17 raisins in each cookie.

How many raisins did he use altogether?

- Work out the missing number.

$6 \times 35 = ___ \times 5$



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# Multiply 4-digits by 2-digits

## Reasoning and Problem Solving

### True or False?

- $5{,}463 \times 18 = 18 \times 5{,}463$
- I can find the answer to $1{,}100 \times 28$ by calculating $1{,}100 \times 30$ and subtracting 2 lots of 1,100
- $702 \times 9 = 701 \times 10$

True

True

False

| 2 | 3 | 4 | 5 | 7 | 8 |
|---|---|---|---|---|---|

Place the digits in the boxes to make the largest product.

| | | | | |
|---|---|---|---|---|
| | □ | □ | □ | □ |
| × | | | □ | □ |
| | | | | |

$$\begin{array}{r} 8432 \\ \times \quad 75 \\ \hline 632000 \end{array}$$



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# Short Division

## Notes and Guidance

Children build on their understanding of dividing up to 4-digits by 1-digit by now dividing by up to 2-digits. They use the short division method and focus on the grouping structure of division. Teachers may encourage children to list multiples of the divisor (number that we are dividing by) to help them solve the division more easily. Children should experience contexts where the answer "4 r 1" means both 4 complete boxes or 5 boxes will be needed.

## Mathematical Talk

In the hundreds column, how many groups of 5 are in 7? Are there are any hundreds remaining? What do we do next?

In the thousands column, there are no groups of three in 1 What do we do?

Why is the context of the question important when deciding how to round the remainders after a division?

## Varied Fluency

Calculate using short division.

| | | | |
|---|---|---|---|
| 5 | 7 | 2 | 5 |

| | | | | |
|---|---|---|---|---|
| 3 | 1 | 9 | 3 | 8 |

| | | | | | |
|---|---|---|---|---|---|
| 1 | 2 | 6 | 0 | 3 | 6 |

3,612 ÷ 14

List the multiples of the divisors to help you calculate.

A limousine company allows 14 people per limousine.

How many limousines are needed for 230 people?

Year 6 has 2,356 pencil crayons for the year.

They put them in bundles, with 12 in each bundle.

How many complete bundles can be made?



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# Short Division

## Reasoning and Problem Solving

Find the missing digits.

$$\begin{array}{r} 041\square r3 \\ 4\overline{)1\square59} \end{array}$$

041 4r3
4)1659

Here are two calculations.

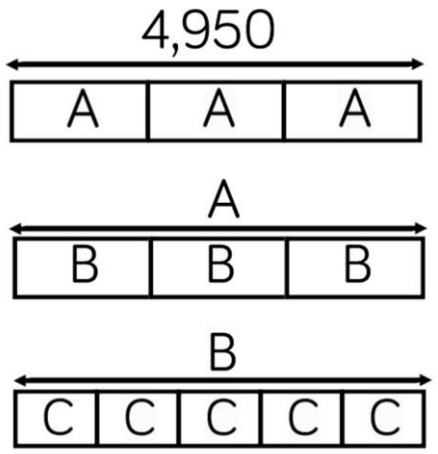
$A = 396 \div 11$

$B = 832 \div 13$

Find the difference between A and B.

$396 \div 11 = 36$
$832 \div 13 = 64$
$64 - 36 = 28$

Work out the value of C.
(The bar models are not drawn to scale)




$4{,}950 \div 3 = 1{,}650$

$1{,}650 \div 3 = 550$

$550 \div 5 = 110$



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# Division using Factors

## Notes and Guidance

Children use their number sense, specifically their knowledge of factors, to be able to see relationships between the dividend (number being divided) and the divisor (number that the dividend is being divided by).

Beginning with multiples of 10 will allow children to see these relationships, before moving to other multiples.

## Mathematical Talk

What is a factor?

How does using factor pairs help us to answer division questions?

Do you notice any patterns?

Does using factor pairs always work?

Is there more than one way to solve a calculation using factor pairs?

What methods can be used to check your working out?

## Varied Fluency

- Calculate 780 ÷ 20

  Now calculate 780 ÷ 10 ÷ 2

  What do you notice? Why does this work?

  Use the same method to calculate 480 ÷ 60

- Use factors to help you calculate.

  4,320 ÷ 15

- Eggs are put into boxes.
  Each box holds 12 eggs.
  A farmer has 648 eggs that need to go in the boxes.

  How many boxes will he fill?





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# Division using Factors

## Reasoning and Problem Solving

Calculate:

- 1,248 ÷ 48
- 1,248 ÷ 24
- 1,248 ÷ 12

What did you do each time? What was your strategy?
What do you notice? Why?

26
52
104
Children should recognise that when the dividend is halved, the answer (quotient) is doubled.

Tommy says,



To calculate 4,320 ÷ 15 I will first divide 4,320 by 5 then divide the answer by 10

Do you agree?
Explain why.

Tommy is wrong: he has partitioned 15 when he should have used factor pairs. He could have used factor pairs 5 and 3 and divided by 5 then 3 (or 3 then 5).

Class 6 are calculating 7,848 ÷ 24

The children decide which factor pairs to use. Here are some of their suggestions:

- 2 and 12
- 1 and 24
- 4 and 6
- 10 and 14

Which will not give them the correct answer? Why?

Use the correct factor pairs to calculate the answer.
Is the answer the same each time?

Which factor pair would be the least efficient to use? Why?

10 and 14 is incorrect because they are not factors of 24 (to get 10 and 14, 24 has been partitioned).

The correct answer is 327

Children should get the same answer using all 3 factor pairs methods.

Using the factor pair of 1 and 24 is the least efficient.



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# Long Division (1)

## Notes and Guidance

Children are introduced to long division as a different method of dividing by a 2-digit number.

They divide 3-digit numbers by a 2-digit number without remainders, starting with a more expanded method (with multiples shown), before progressing to the more formal long division method.

## Mathematical Talk

How can we use multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided).

## Varied Fluency

| | | 0 | 3 | 6 | |
|---|---|---|---|---|---|
| 1 | 2 | 4 | 3 | 2 | (×30) |
| | – | 3 | 6 | 0 | |
| | | | 7 | 2 | (×6) |
| | – | | 7 | 2 | |
| | | | | 0 | |

Multiples of 12:

$12 \times 1 = 12$
$12 \times 2 = 24$
$12 \times 3 = 36$
$12 \times 4 = 48$
$12 \times 5 = 60$
$12 \times 6 = 72$
$12 \times 7 = 84$
$12 \times 8 = 96$
$12 \times 7 = 108$
$12 \times 10 = 120$

Use this method to calculate:

765 ÷ 17 450 ÷ 15 702 ÷ 18

| | | 0 | 3 | 6 | |
|---|---|---|---|---|---|
| 1 | 2 | 4 | 3 | 2 | |
| | – | 3 | 6 | ↓ | |
| | | | 7 | 2 | |
| | – | | 7 | 2 | |
| | | | | 0 | |

Use the long division method to calculate:

836 ÷ 11
798 ÷ 14
608 ÷ 19



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# Long Division (1)

## Reasoning and Problem Solving

### Odd One Out

Which is the odd one out?
Explain your answer.

512 ÷ 16

672 ÷ 21

792 ÷ 24

792 ÷ 24 = 33 so this is the odd one out as the other two give an answer of 32

### Spot the Mistake

855 ÷ 15 =

| | | | | | | |
|---|---|---|---|---|---|---|
| | | 0 | 5 | 1 | 0 | |
| 1 | 5 | 8 | 5 | 5 | | |
| | – | 7 | 5 | | ( × | 4) |
| | | 1 | 0 | 5 | | |
| | – | 1 | 0 | 5 | ( × | 10) |
| | | | | 0 | | |

The mistake is that 105 ÷ 15 is not equal to 10

105 ÷ 15 = 7 so the answer to the calculation is 57



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# Long Division (2)

## Notes and Guidance

Building on using long division with 3-digit numbers, children divide 4-digit numbers by 2-digits using the long division method.

They use their knowledge of multiples and multiplying and dividing by 10 and 100 to calculate more efficiently.

## Mathematical Talk

How can we use multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided).

## Varied Fluency

Here is a division method.

| | 0 | 4 | 8 | 9 | |
|---|---|---|---|---|---|
| 15 | 7 | 3 | 3 | 5 | |
| – | 6 | 0 | 0 | 0 | (×400) |
| | 1 | 3 | 3 | 5 | |
| – | 1 | 2 | 0 | 0 | (×80) |
| | | 1 | 3 | 5 | |
| – | | 1 | 3 | 5 | (×9) |
| | | | | 0 | |

Use this method to calculate:

2,208 ÷ 16 1,755 ÷ 45 1,536 ÷ 16

There are 1,989 footballers in a tournament.
Each team has 11 players and 2 substitutes.
How many teams are there in the tournament?



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# Long Division (2)

## Reasoning and Problem Solving

Which calculation is harder?

$1{,}950 \div 13$

$1{,}950 \div 15$

Explain why.

Dividing by 13 is harder because 13 is prime so we cannot use factor knowledge to factorise it into smaller parts. The 13 times table is harder than the 15 times table because the 15 times table is related to the 5 times table whereas the 13 times table is not related to a more common times table (because 13 is prime).

$6{,}120 \div 17 = 360$

Explain how to use this fact to find



$6{,}480 \div$  $= 360$

6,480 is 360 more than 6,120, so there is 1 group of 360 more.

Therefore, there are 18 groups of 360, so the answer is 18



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# Long Division (3)

## Notes and Guidance

Children now divide using long division where answers have remainders. After dividing, they check that the remainder is smaller than the divisor.

Children start to understand how to interpret the remainder e.g. **380 ÷ 12 = 31 r 8 could mean 31 full packs, or 32 packs needed depending on context.**

## Mathematical Talk

How can we use multiples to help us divide?

What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over?

Why are we subtracting the totals from the dividend (starting number)?

Why is the context of the question important when deciding how to round the remainders after a division?

## Varied Fluency

Tommy uses this method to calculate 372 divided by 15
He has used his knowledge of multiples to help.

| | | | 2 | 4 | r | 1 | 2 |
|---|---|---|---|---|---|---|---|
| 1 | 5 | 3 | 7 | 2 | | | |
| | – | 3 | 0 | 0 | | | |
| | | | 7 | 2 | | | |
| | – | | 6 | 0 | | | |
| | | | 1 | 2 | | | |

$1 \times 15 = 15$
$2 \times 15 = 30$
$3 \times 15 = 45$
$4 \times 15 = 60$
$5 \times 15 = 75$
$10 \times 15 = 150$

Use this method to calculate:

$271 \div 17$ $623 \div 21$ $842 \div 32$

A school needs to buy 380 biscuits for parents' evening.
Biscuits are sold in packs of 12

How many packets will the school need to buy?



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# Long Division (3)

## Reasoning and Problem Solving

Here are two calculation cards.

A = 396 ÷ 11

B = 832 ÷ 11

Whitney thinks there won't be a remainder for either calculation because 396 and 832 are both multiples of 11

Rosie disagrees, she has done the written calculations and says **one** of them has a remainder.

Who is correct? Explain your answer.

Rosie is correct because 832 is not a multiple of 11

396 ÷ 11 = 36

832 ÷ 11 = 75 r 7

576 children and 32 adults need transport for a school trip.
A coach holds 55 people.



Dora: We need 10 coaches.



Eva: We need 11 coaches.



Alex: We need 12 coaches.

Who is correct? Explain how you know.

How many spare seats will there be?

Alex is correct.

There are 608 people altogether, 608 ÷ 55 = 11 r 3, so 12 coaches are needed.

On 12 coaches there will be 660 seats, because 55 × 12 = 660
660 – 608 = 52 spare seats.



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# Long Division (4)

## Notes and Guidance

Children now divide four-digit numbers using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that it is not applicable.

## Mathematical Talk

How can we use multiples to help us divide?

What happens if we cannot divide the ones exactly by the divisor? How do we show what is left over?

Why are we subtracting the totals from the dividend (starting number)? This question supports children to see division as repeated subtraction.

Does the remainder need to be rounded up or down?

## Varied Fluency

Amir used this method to calculate 1,426 divided by 13

| | | | 1 | 0 | 9 | r | 9 | |
|---|---|---|---|---|---|---|---|---|
| 1 | 3 | 1 | 4 | 2 | 6 | | | |
| | – | 1 | 3 | 0 | 0 | | | (× 100) |
| | | | 1 | 2 | 6 | | | |
| | – | | 1 | 1 | 7 | | | (×9) |
| | | | | | 9 | | | |

Use this method to calculate:

2,637 ÷ 16 4,453 ÷ 22 4,203 ÷ 18

A large bakery produces 7,849 biscuits in a day which are packed in boxes.
Each box holds 64 biscuits.

How many boxes are needed so all the biscuits are in a box?



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# Long Division (4)

## Reasoning and Problem Solving

Class 6 are calculating three thousand, six hundred and thirty-three divided by twelve.

Rosie says that she knows there will be a remainder without calculating.

Is she correct?
Explain your answer.

What is the remainder?

Rosie is correct because 3,633 is odd and 12 is even, and all multiples of 12 are even because 12 is even.

3,633 ÷ 12 = 302 r 9, so the remainder is 9

Which numbers up to 20 can 4,236 be divided by without having a remainder?

What do you notice about all the numbers?

1, 2, 3, 4, 6, 12

They are all factors of 12



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# Common Factors

## Notes and Guidance

Children find the common factors of two numbers.

Some children may still need to use arrays and other representations at this stage but mental methods and knowledge of multiples should be encouraged.

They can show their results using Venn diagrams and tables.

## Mathematical Talk

How do you know you have found all the factors of a given number?

Have you used a systematic approach?

Can you explain your system to a partner?

How does a Venn diagram show common factors?

Where are the common factors?

## Varied Fluency

Find the common factors of each pair of numbers.

24 and 36

20 and 30

28 and 45

Which number's factors make it the odd one out?

12, 30, 54, 42, 32, 48

Can you explain why?

Two numbers have common factors of 4 and 9
What could the numbers be?



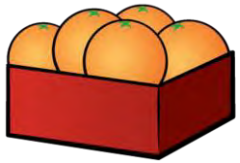
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# Common Factors

## Reasoning and Problem Solving

There are 49 pears and 56 oranges.





They need to be put into baskets of pears and baskets of oranges with an equal number of fruit in each basket.

Amir says,



There will be 8 pieces of fruit in each basket.

Jack says,

There will be 7 pieces of fruit in each basket.



Who is correct? Explain how you know.

Jack is correct. There will be seven pieces of fruit in each basket because 7 is a common factor of 49 and 56

Tommy has two pieces of string.

One is 160 cm long and the other is 200 cm long.

He cuts them into pieces of equal length.

What are the possible lengths the pieces of string could be?

The possible lengths are: 2, 4, 5, 8, 10, 20 and 40 cm.

Dora has 32 football cards that she is giving away to his friends.

She shares them equally between her friends.

How many friends could Dora have?

Dora could have 1, 2, 4, 8, 16 or 32 friends.



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# Common Multiples

## Notes and Guidance

Building on knowledge of multiples, children find common multiples of numbers. They should continue to use visual representations to support their thinking.

They also use abstract methods to calculate multiples, including using numbers outside of those known in times table facts.

## Mathematical Talk

Is the lowest common multiple of a pair of numbers always the product of them?

Can you think of any strategies to work out the lowest common multiples of different numbers?

When do numbers have common multiples that are lower than their product?

## Varied Fluency

- On a 100 square, shade the first 5 multiples of 7 and then the first 8 multiples of 5

  What common multiple of 7 and 5 do you find?

  Use this number to find other common multiples of 7 and 5

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

- List 5 common multiples of 4 and 3

- Alex and Eva play football at the same local football pitches. Alex plays every 4 days and Eva plays every 6 days.

  They both played football today.

  After a fortnight, how many times will they have played football on the same day?

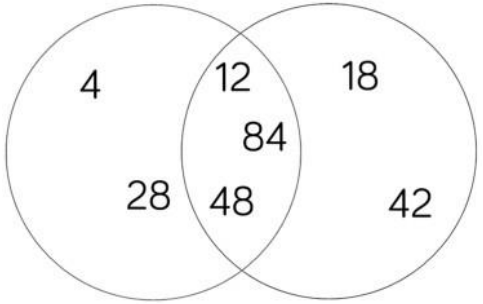


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# Common Multiples

## Reasoning and Problem Solving

Work out the headings for the Venn diagram.




Add in one more number to each section.

Can you find a square number that will go in the middle section of the Venn diagram?

Multiples of 4
Multiples of 6

144 is a square number that can go in the middle.

Annie is double her sister's age.

They are both older than 20 but younger than 50

Their ages are both multiples of 7

What are their ages?

Annie is 42 and her sister is 21

A train starts running from Leeds to York at 7am.
The last train leaves at midnight.

Platform 1 has a train leaving from it every 12 minutes.
Platform 2 has one leaving from it every 5 minutes.

How many times in the day would there be a train leaving from both platforms at the same time?

18 times



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# Primes to 100

## Notes and Guidance

Building on their learning in year 5, children should know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

They should be able to use their understanding of prime numbers to work out whether or not numbers up to 100 are prime. Using primes, they break a number down into its prime factors.

## Mathematical Talk

What is a prime number?

What is a composite number?

How many factors does a prime number have?

Are all prime numbers odd?

Why is 1 not a prime number?

Why is 2 a prime number?

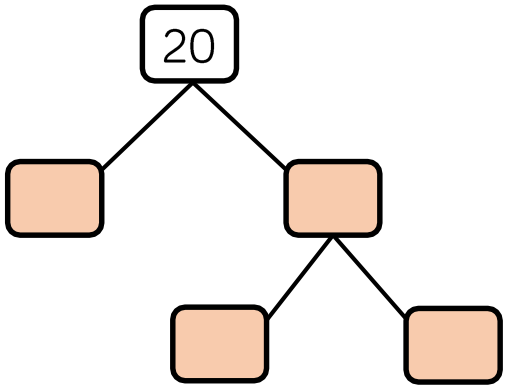
## Varied Fluency

- List all of the prime numbers between 10 and 30

- The sum of two prime numbers is 36

  What are the numbers?

- All numbers can be broken down into prime factors.
  A prime factor tree can help us find them.
  Complete the prime factor tree for 20






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# Primes to 100

## Reasoning and Problem Solving

Use the clues to work out the number.

- It is greater than 10
- It is an odd number
- It is not a prime number
- It is less than 25
- It is a factor of 60

15

Shade in the multiples of 6 on a 100 square.

What do you notice about the numbers either side of every multiple of 6?

Eva says,



I noticed there is always a prime number next to a multiple of 6

Is she correct?

Both numbers are always odd.

Yes, Eva is correct because at least one of the numbers either side of a multiple of 6 is always prime for numbers up to 100



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# Square & Cube Numbers

## Notes and Guidance

Children have identified square and cube numbers previously and now explore the relationship between them, and solve problems involving them.
They need to experience sorting the numbers into different diagrams and look for patterns and relationships. They explore general statements regarding square and cube numbers. This step is a good opportunity to practise efficient mental methods of calculation.

## Mathematical Talk

What do you notice about the sequence of square numbers?

What do you notice about the sequence of cube numbers?

Explore the pattern of the difference between the numbers.

## Varied Fluency

Use <, > or = to make the statements correct.

3 cubed ◯ 4 squared

8 squared ◯ 4 cubed

11 squared ◯ 5 cubed

This table shows square and cube numbers. Complete the table.
Explain the relationships you can see between the numbers.

| | | | | | |
|---|---|---|---|---|---|
| | | 1 | | | 1 |
| | | | | | 8 |
| | $3 \times 3$ | | $3^3$ | | 27 |
| | $4 \times 4$ | | | $4 \times 4 \times 4$ | |
| | | 25 | $5^3$ | | |
| | | | | $6 \times 6 \times 6$ | |
| | | | | | |
| $8^2$ | | | | | |

___ + 35 = 99

210 − ___ = 41

Which square numbers are missing from the calculations?



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# Square & Cube Numbers

## Reasoning and Problem Solving

Place 5 odd and 5 even numbers in the table.

| | Not Cubed | Cubed |
|---|---|---|
| Over 100 | | |
| 100 or less | | |

Possible cube numbers to use:

1, 8, 27, 64, 125, 216, 343, 512, 729, 1,000

Jack says,



The smallest number that is both a square number and a cube number is 64

Do you agree with Jack? Explain why you agree or disagree.

Jack is incorrect. 1 is the smallest number that is both a square number ($1^2 = 1$) and cube number ($1^3 = 1$).

Shade in all the square numbers on a 100 square.

Now shade in multiples of 4

What do you notice?

Square numbers are always either a multiple of 4 or 1 more than a multiple of 4

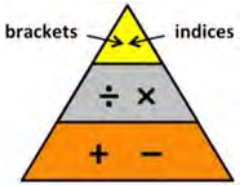


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# Order of Operations

## Notes and Guidance

Children will look at different operations within a calculation and consider how the order of operations affects the answer.
Children will learn that, in mixed operation calculations, calculations are not carried out from left to right.
Children learn the convention that when there is no operation sign written this means multiply e.g. $4(2 + 1)$ means $4 \times (2 + 1)$. This image is useful when teaching the order of operations.




## Mathematical Talk

Does it make a difference if you change the order in a mixed operation calculation?

What would happen if we did not use the brackets?

Would the answer be correct?

Why?

## Varied Fluency

- Alex has 7 bags with 5 sweets in each bag.
She adds one more sweet to each bag.
Which calculation will work out how many sweets she now has in total? Explain your answer.

$$7 \times (5 + 1)$$

$$7 \times 5 + 1$$

- Teddy has completed this calculation and got an answer of 5

$$14 - 4 \times 2 \div 4 = 5$$

Explain and correct his error.

- Add brackets and missing numbers to make the calculations correct.

$$6 + ___ \times 5 = 30$$

$$25 - 6 \times ___ = 38$$



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# Order of Operations

## Reasoning and Problem Solving

### Countdown

Big numbers: 25, 50, 75, 100

Small numbers: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Children randomly select 6 numbers.

Reveal a target number.

Children aim to make the target number ensuring they can write it as a single calculation using order of operations.

Write different number sentences using the digits 3, 4, 5 and 8 before the equals sign that use:

- One operation
- Two operations with no brackets
- Two operations with brackets

Possible solutions:

$58 - 34 = 24$

$58 + 3 \times 4 = 60$

$5(8 - 3) + 4 = 29$



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# Mental Calculations

## Notes and Guidance

We have included this small step separately to ensure that teachers emphasise this important skill. Discussions with children around efficient mental calculations and sensible estimations need to run through all steps.

Sometimes children are too quick to move to computational methods, when more efficient mental strategies should be used.

## Mathematical Talk

Is there an easy and quick way to do this?

Can you use known facts to answer the problem?

Can you use rounding?

Does the solution need an exact answer?

How does knowing the approximate answer help with the calculation?

## Varied Fluency

How could you change the order of these calculations to be able to perform them mentally?

$50 \times 16 \times 2$

$30 \times 12 \times 2$

$4 \times 17 \times 25$

Mo wants to buy a t-shirt for £9.99, socks for £1.49 and a belt for £8.99
He has £22 in his wallet.
How could he quickly check if he has enough money?




What number do you estimate is shown by arrow B when:

- A = 0 and C = 1,000
- A = 30 and C = 150
- A = −7 and C = 17
- A = 1 and C = 2
- A = 1,000 and C = 100,000



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# Mental Calculations

## Reasoning and Problem Solving

Class 6 are calculating the total of 3,912 and 3,888

Alex says,



We can just double 3,900

Is Alex correct? Explain.

Alex is correct because 3,912 is 12 more than 3,900 and 3,888 is 12 less than 3,900

3,900 × 2 = 7,800

2,000 – 1,287

Here are three different strategies for this subtraction calculation:



I used the column method.

Dora



I used my number bonds from 87 to 100 then from 1,300 to 2,000

Tommy



I subtracted one from each number and then used the column method.

Jack

Whose method is most efficient?

Children share their ideas. Discuss how Dora's method is inefficient for this calculation because of the need to make multiple exchanges.

Jack's method is known as the 'constant difference' method and avoids exchanging.



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# Reason from Known Facts

## Notes and Guidance

Children should use known facts from one calculation to determine the answer of another similar calculation without starting afresh.

They should use reasoning and apply their understanding of commutativity and inverse operations.

## Mathematical Talk

What is the inverse?

When do you use the inverse?

How can we use multiplication/division facts to help us answer similar questions?

## Varied Fluency

Complete.

$70 \div ___ = 7$ $3.5 \times 10 = ___$

$70 \div ___ = 3.5$ $___ = 3.5 \times 20$

$70 \div ___ = 14$ $___ = 3.5 \times 2$

Make a similar set of calculations using $90 \div 2 = 45$

$5{,}138 \div 14 = 367$

Use this to calculate $15 \times 367$

$14 \times 8 = 112$

Use this to calculate:

- $1.4 \times 8$
- $9 \times 14$



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# Reason from Known Facts

## Reasoning and Problem Solving

$$3{,}565 + 2{,}250 = 5{,}815$$

Use this calculation to decide if the following calculations are true or false.

### True or False?

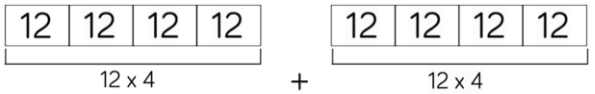
| | |
|---|---|
| 4,565 + 1,250 = 5,815 | True |
| 5,815 – 2,250 = 3,565 | True |
| 4,815 – 2,565 = 2,250 | True |
| 3,595 + 2,220 = 5,845 | False |

Which calculations will give an answer that is the same as the product of 12 and 8?

$$3 \times 4 \times 8$$

$$12 \times 4 \times 2$$

$$2 \times 10 \times 8$$




The product of 12 and 8 is 96

The 1st and 2nd calculations give an answer of 96
In the 1st calculation 12 has been factorised into 3 and 4, and in the 2nd calculation 8 has been factorised into 4 and 2

The third calculation gives an answer of 160



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Autumn - Block 3

# Fractions

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# Overview

## Small Steps

1. Simplify fractions
2. Fractions on a number line
3. Compare and order (denominator)
4. Compare and order (numerator)
5. Add and subtract fractions (1)
6. Add and subtract fractions (2)
7. Add fractions
8. Subtract fractions
9. Mixed addition and subtraction
10. Multiply fractions by integers
11. Multiply fractions by fractions
12. Divide fractions by integers (1)
13. Divide fractions by integers (2)
14. Four rules with fractions
15. Fraction of an amount
16. Fraction of an amount – find the whole

## NC Objectives

Use common factors to simplify fractions; use common multiples to express fractions in the same denomination.

Compare and order fractions, including fractions $> 1$

Generate and describe linear number sequences (with fractions)

Add and subtract fractions with different denominations and mixed numbers, using the concept of equivalent fractions.
Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]

Divide proper fractions by whole numbers [for example $\frac{1}{3} \div 2 = \frac{1}{6}$]
Associate a fraction with division and calculate decimal fraction equivalents [ for example, 0.375] for a simple fraction [for example $\frac{1}{8}$]

Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.



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# Simplify Fractions

## Notes and Guidance

Children use their understanding of the highest common factor to simplify fractions, building on their knowledge of equivalent fractions in earlier years.
Children apply their understanding when calculating with fractions and simplifying their answers. Encourage children to use pictorial representations to support simplifying e.g. a fraction wall.

## Mathematical Talk

Can you make a list of the factors for each number?
Which numbers appear in both lists? What do we call these (common factors)?
What is the highest common factor of the numerator and denominator?
Is a simplified fraction always equivalent to the original fraction? Why?
If the HCF of the numerator and denominator is 1, can it be simplified?

## Varied Fluency

Alex is simplifying $\frac{8}{12}$ by dividing the numerator and denominator by their highest common factor.



Factors of 8: 1 , 2 , **4** , 8
Factors of 12: 1 , 2 , 3 , **4** , 6 , 12
4 is the highest common factor.

$$\frac{8}{12} \overset{\div 4}{=} \frac{2}{3}$$

Use Alex's method to simplify these fractions:

$$\frac{6}{9} \quad \frac{6}{18} \quad \frac{10}{18} \quad \frac{10}{15} \quad \frac{15}{50}$$

Mo has 3 boxes of chocolates. 2 boxes are full and one box is $\frac{4}{10}$ full.



To simplify $2\frac{4}{10}$, keep the whole number the same and simplify the fraction. $\frac{4}{10}$ simplifies to $\frac{2}{5}$

$$2\frac{4}{10} = 2\frac{2}{5}$$

Use Mo's method to simplify:

$$3\frac{4}{8}, \quad 5\frac{9}{21}, \quad 2\frac{7}{21}, \quad \frac{32}{10}, \quad \frac{32}{6}$$



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# Simplify Fractions

## Reasoning and Problem Solving

Find the total of the fractions.
Give your answer in its simplest form.

$\frac{5}{9}+\frac{1}{9}=$ $\frac{5}{9}+\frac{3}{9}=$ $\frac{5}{9}+\frac{7}{9}=$

Do all the answers need simplifying?
Explain why.

$\frac{5}{9}+\frac{1}{9}=\frac{6}{9}=\frac{2}{3}$

$\frac{5}{9}+\frac{3}{9}=\frac{8}{9}$

$\frac{5}{9}+\frac{7}{9}=1\frac{3}{9}=1\frac{1}{3}$

$\frac{8}{9}$ does not need simplifying because the HCF of 8 and 9 is 1

Tommy is simplifying $4\frac{12}{16}$

$$4\frac{12}{16}=1\frac{3}{4}$$

Explain Tommy's mistake.

Tommy has divided the whole number by 4 instead of just simplifying $\frac{12}{16}$ by dividing the numerator and denominator by 4

Sort the fractions into the table.

| Simplifies to $\frac{1}{2}$ | Simplifies to $\frac{1}{3}$ | Simplifies to $\frac{1}{4}$ |
|---|---|---|
| | | |

$\frac{5}{15}$ $\frac{2}{4}$ $\frac{4}{16}$ $\frac{8}{16}$ $\frac{5}{10}$ $\frac{3}{9}$ $\frac{6}{12}$ $\frac{2}{8}$

Can you see any patterns between the numbers in each column?
What is the relationship between the numerators and denominators?
Can you add three more fractions to each column?

Complete the sentence to describe the patterns:
When a fraction is equivalent to ____, the numerator is _____ the denominator.

Simplifies to $\frac{1}{2}$ - $\frac{2}{4}$, $\frac{8}{16}$, $\frac{5}{10}$, $\frac{6}{12}$
Simplifies to $\frac{1}{3}$ - $\frac{5}{15}$, $\frac{3}{9}$
Simplifies to $\frac{1}{4}$ - $\frac{4}{16}$, $\frac{2}{8}$

When a fraction is equivalent to a half, the numerator is half the denominator.
Children could also discuss the denominator being double the numerator.
Repeat for $\frac{1}{3}$ and $\frac{1}{4}$



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# Fractions on a Number Line

## Notes and Guidance

Children count forwards and backwards in fractions. They compare and order fractions with the same denominator or denominators that are multiples of the same number.
Encourage children to draw extra intervals on the number lines to support them to place the fractions more accurately.
Children use the divisions on the number line to support them in finding the difference between fractions.

## Mathematical Talk

Which numbers do I say when I count in eighths and when I count in quarters?

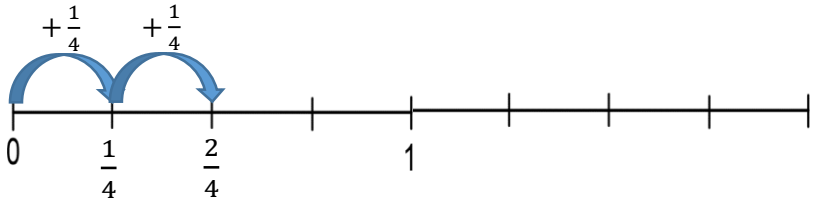
Can you estimate where the fractions will be on the number line?

Can you divide the number line into more intervals to place the fractions more accurately?

How can you find the difference between the fractions?

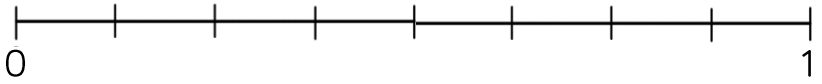
## Varied Fluency

Jack is counting in quarters. He writes each number he says on a number line.
Complete Jack's number line.




Can you simplify any of the fractions on the number line?
Can you count forward in eighths? How would the number line change?

Place $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}, \frac{5}{8}, \frac{7}{8}$ and $\frac{3}{16}$ on the number line.




Which fractions were the easiest to place?
Which fractions were the hardest to place?
Which fraction is the largest? Which fraction is the smallest?
What is the difference between the largest and smallest fraction?



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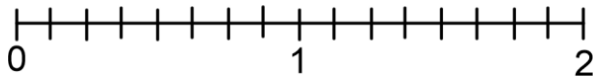
# Fractions on a Number Line

## Reasoning and Problem Solving

Rosie is counting backwards in fifths.
She starts at $3\frac{2}{5}$ and counts back nine fifths.
What number does Rosie end on?
Show this on a number line.

Rosie ends on $1\frac{3}{5}$

How many ways can you show a difference of one quarter on the number line?




Various answers available.

Plot the sequences on a number line.

$3\frac{1}{2}, 4, 4\frac{1}{2}, 5, 5\frac{1}{2}, 6$

$\frac{13}{4}, \frac{15}{4}, \frac{17}{4}, \frac{19}{4}, \frac{21}{4}, \frac{23}{4}$

$5\frac{5}{8}, 5\frac{1}{8}, 4\frac{5}{8}, 4\frac{1}{8}, 3\frac{5}{8}, 3\frac{1}{8}$

$3\frac{1}{8}, 3\frac{3}{8}, 3\frac{5}{8}, 3\frac{7}{8}, 4\frac{1}{8}, 4\frac{3}{8}$

Which sequence is the odd one out?
Explain why.

Can you think of a reason why each of the sequences could be the odd one out?

Children may choose different sequences for different reasons.
First sequence: the only one containing 6 or it is the only one containing whole numbers.
Second sequence: only one using improper fractions
Third sequence: the only one going backwards.
Fourth sequence: only one not counting in halves.



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# Compare & Order (Denominator)

## Notes and Guidance

Children use their knowledge of equivalent fractions to compare fractions where the denominators are not multiples of the same number.
They find the lowest common multiple of the denominators in order to find equivalent fractions with the same denominators.
Children then compare the numerators to find the larger or smaller fraction. Encourage children to also use their number sense to visualise the size of the fractions before converting.

## Mathematical Talk

When I know the lowest common multiple, how do I know what to multiply the numerator and denominator by to find the correct equivalent fraction?
How is comparing mixed numbers different to comparing proper fractions? Do I need to compare the whole numbers? Why? If the whole numbers are the same, what do I do?
Can you plot the fractions on a number line to estimate which is the smallest? Which fractions are larger/smaller than a half? How does this help me order the fractions?

## Varied Fluency

Use the bar models to compare $\frac{3}{4}$ and $\frac{2}{3}$



____ is greater than ____

____ is less than ____

Dora is comparing $\frac{5}{6}$ and $\frac{3}{4}$ by finding the lowest common multiple of the denominators.



Multiples of 6: 6, 12, 18, 24
Multiples of 4: 4, 8, 12, 16,
12 is the LCM of 4 and 6

$\frac{5}{6} = \frac{10}{12}$ $\frac{3}{4} = \frac{9}{12}$

$\frac{10}{12} > \frac{9}{12}$

Use Dora's method to compare the fractions.

$\frac{4}{5} \bigcirc \frac{3}{4}$ $\frac{3}{5} \bigcirc \frac{4}{7}$ $\frac{3}{4} \bigcirc \frac{7}{10}$ $2\frac{2}{5} \bigcirc 2\frac{3}{8}$

Order the fractions in descending order.

$\frac{3}{8}, \frac{11}{20}, \frac{1}{2}, \frac{2}{5}, \frac{3}{4}, \frac{7}{10}$

Which fraction is the greatest?
Which fraction is the smallest?

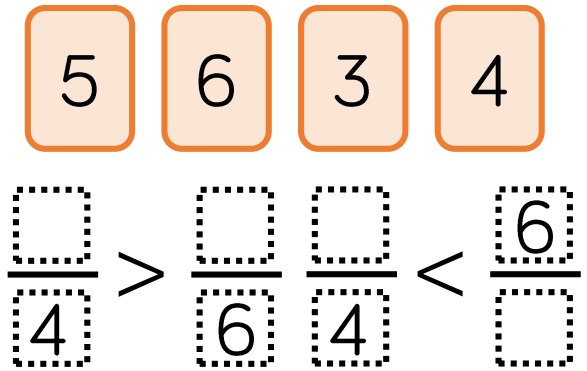


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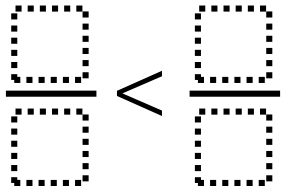
# Compare & Order (Denominator)

## Reasoning and Problem Solving

Use the digit cards to complete the statements.




Find three examples of ways you could complete the statement.




Can one of your ways include an improper fraction?

$\frac{5}{4} > \frac{3}{6}$

$\frac{3}{4} < \frac{6}{5}$ or $\frac{5}{4} < \frac{6}{3}$

$\frac{3}{5} < \frac{6}{4}$
$\frac{3}{4} < \frac{6}{5}$
$\frac{4}{5} < \frac{6}{3}$

More answers available.

Teddy is comparing $\frac{3}{8}$ and $\frac{5}{12}$



To find the lowest common multiple, I will multiply 8 and 12 together.
$8 \times 12 = 96$
I will use a common denominator of 96

Is Teddy correct?
Explain why.

Teddy is incorrect because the LCM of 8 and 12 is 24
96 is a common multiple so he would still compare the fractions correctly but it is not the most efficient method.



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# Compare & Order (Numerator)

## Notes and Guidance

Building on their prior knowledge of comparing unit fractions, children look at comparing fractions by finding a common numerator. They focus on the idea that when the numerators are the same, the larger the denominator, the smaller the fraction.
Children consider the most efficient method when comparing fractions and decide whether to find common numerators or common denominators.

## Mathematical Talk

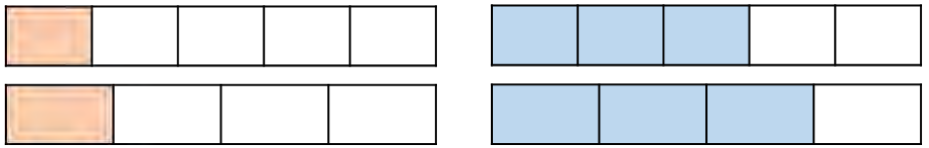
What's the same and what's different about the fractions on the bar models? How can we compare them? Can you use the words greatest and smallest to complete the sentences?

Do you need to change one or both numerators? Why?

How can you decide whether to find a common numerator or denominator?

## Varied Fluency

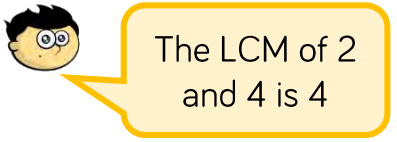
Compare the fractions.



$\frac{1}{5} \bigcirc \frac{1}{4}$ $\frac{1}{5} \bigcirc \frac{3}{5}$ $\frac{1}{4} \bigcirc \frac{3}{4}$ $\frac{3}{5} \bigcirc \frac{3}{4}$

When the denominators are the same, the _____ the numerator, the _____ the fraction.
When the numerators are the same, the _____ the denominator, the _____ the fraction.

Jack is comparing $\frac{2}{5}$ and $\frac{4}{7}$ by finding the LCM of the numerators.




$\frac{2}{5} = \frac{4}{10}$ $\frac{4}{10} < \frac{4}{7}$

Use Jack's method to compare the fractions.

$\frac{3}{5} \bigcirc \frac{12}{17}$ $\frac{6}{11} \bigcirc \frac{3}{5}$ $\frac{5}{9} \bigcirc \frac{4}{7}$ $\frac{8}{5} \bigcirc \frac{12}{7}$



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# Compare & Order (Numerator)

## Reasoning and Problem Solving

Mo is comparing the fractions $\frac{3}{7}$ and $\frac{6}{11}$

He wants to find a common denominator.

Explain whether you think this is the most effective strategy.

This is not the most effective strategy because both denominators are prime. He could find a common numerator by changing $\frac{3}{7}$ into $\frac{6}{14}$ and comparing them by using the rule 'when the numerator is the same, the smaller the denominator, the bigger the fraction' $\frac{6}{11}$ is bigger.

Two different pieces of wood have had a fraction chopped off.

Here are the pieces now, with the fraction that is left.




Which piece of wood was the longest to begin with?

Explain your answer.

Can you explain your method?

The second piece was longer because $\frac{1}{4}$ is greater than $\frac{1}{6}$. Children can explain their methods and how they compared one quarter and one sixth.



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# Add & Subtract Fractions (1)

## Notes and Guidance

Children add and subtract fractions within 1 where the denominators are multiples of the same number. Encourage children to find the lowest common multiple in order to find a common denominator. Ensure children are confident with the understanding of adding and subtracting fractions with the same denominator. Bar models can support this, showing children that the denominators stay the same whilst the numerators are added or subtracted.

## Mathematical Talk

If the denominators are different, when we are adding or subtracting fractions, what do we need to do? Why?

How does finding the lowest common multiple help to find a common denominator?

Can you use a bar model to represent Eva's tin of paint? On which day did Eva use the most paint? On which day did Eva use the least paint? How much more paint did Eva use on Friday than Saturday?

## Varied Fluency

- Whitney is calculating $\frac{5}{8} + \frac{3}{16}$
  She finds the lowest common multiple of 8 and 16 to find a common denominator.
  LCM of 8 and 16 is 16 $\quad \frac{5}{8} = \frac{10}{16} \quad \frac{10}{16} + \frac{3}{16} = \frac{13}{16}$
  Use this method to calculate:
  $\frac{1}{3} + \frac{2}{9} =$ $\quad \frac{3}{7} + \frac{7}{21} =$ $\quad \frac{8}{15} + \frac{1}{5} =$ $\quad \frac{3}{16} + \frac{3}{8} + \frac{1}{4} =$

- Find a common denominator for each pair of fractions by using the lowest common multiple. Subtract the smaller fraction from the larger fraction in each pair.
  $\frac{3}{4}, \frac{5}{8}$ $\quad \frac{7}{12}, \frac{1}{3}$ $\quad \frac{11}{16}, \frac{3}{4}$ $\quad \frac{14}{15}, \frac{2}{5}$ $\quad \frac{8}{9}, \frac{1}{3}$

- Eva has a full tin of paint. She uses $\frac{1}{3}$ of the tin on Friday, $\frac{1}{21}$ on Saturday and $\frac{2}{7}$ on Sunday. How much paint does she have left?



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# Add & Subtract Fractions (1)

## Reasoning and Problem Solving

Use the same digit in both boxes to complete the calculation.
Is there more than one way to do it?

$$\frac{\square}{20} + \frac{1}{\square} = \frac{9}{20}$$

$\frac{4}{20} + \frac{1}{4} = \frac{9}{20}$

$\frac{5}{20} + \frac{1}{5} = \frac{9}{20}$

Dexter subtracted $\frac{3}{5}$ from a fraction and his answer was $\frac{8}{45}$

What fraction did he subtract $\frac{3}{5}$ from?
Give your answer in its simplest form.

$\frac{8}{45} + \frac{3}{5} = \frac{8}{45} + \frac{27}{45}$

$\frac{8}{45} + \frac{27}{45} = \frac{35}{45} = \frac{7}{9}$

Dexter subtracted $\frac{3}{5}$ from $\frac{7}{9}$

Alex is adding fractions.

$$\frac{3}{5} + \frac{1}{15} = \frac{4}{20} = \frac{1}{5}$$

Do you agree with her?
Explain your answer.

Alex is wrong because she has added the numerators and the denominators rather than finding a common denominator.
It should be

$\frac{9}{15} + \frac{1}{15} = \frac{10}{15} = \frac{2}{3}$



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# Add & Subtract Fractions (2)

## Notes and Guidance

Children add and subtract fractions where the denominators are not multiples of the same number. They continue to find the lowest common multiple, but now need to find equivalent fractions for both fractions in the calculation to find a common denominator.
When the denominators are not multiples of the same number, support children to notice that we need to multiply the denominators together in order to find the LCM.

## Mathematical Talk

What is the same about all the subtractions? ($\frac{3}{4}$)

What do you notice about the LCM of all the denominators?

Which of the subtractions has the biggest difference? Explain how you know. Can you order the differences in ascending order?

How can we find the LCM of three numbers? Do we multiply them together? Is 120 the LCM of 4, 5 and 6?

## Varied Fluency

Amir is calculating $\frac{7}{9} - \frac{1}{2}$

He finds the lowest common multiple of 9 and 2
LCM of 9 and 2 is 18

$$\frac{7}{9} - \frac{1}{2} = \frac{14}{18} - \frac{9}{18} = \frac{5}{18}$$

Use this method to calculate:

$\frac{3}{4} - \frac{1}{3} =$ $\frac{3}{4} - \frac{3}{5} =$ $\frac{3}{4} - \frac{2}{7} =$ $\frac{3}{4} - \frac{7}{11} =$

Eva has a bag of carrots weighing $\frac{3}{4}$ kg and a bag of potatoes weighing $\frac{2}{5}$ kg. She is calculating how much they weigh altogether.



The LCM of 4 and 5 is 20. I will convert the fractions to twentieths.

$\frac{3}{4} + \frac{2}{5} = \frac{15}{20} + \frac{8}{20} = \frac{23}{20} = 1\frac{3}{20}$ kg

Use this method to calculate:

$\frac{1}{4} + \frac{2}{5} =$ $\frac{7}{8} + \frac{1}{3} =$ $\frac{5}{6} + \frac{5}{7} =$ $\frac{13}{20} + \frac{2}{3} =$

On Friday, Ron walks $\frac{5}{6}$ km to school, $\frac{3}{4}$ km to the shops and $\frac{4}{5}$ km home. How far does he walk altogether?



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# Add & Subtract Fractions (2)

## Reasoning and Problem Solving

A car is travelling from Halifax to Brighton.
In the morning, it completes $\frac{2}{3}$ of the journey.
In the afternoon, it completes $\frac{1}{5}$ of the journey.
What fraction of the journey has been travelled altogether?
What fraction of the journey is left to travel?

The car has travelled $\frac{13}{15}$ of the journey altogether.

There is $\frac{2}{15}$ of the journey left to travel.

If the journey is 270 miles, how far did the car travel in the morning?
How far did the car travel in the afternoon?
How far does the car have left to travel?



The car travelled 180 miles in the morning.
The car travelled 54 miles in the afternoon.
The car has 36 miles left to travel.

Mr and Mrs Rose and knitting scarves.
Mr Rose's scarf is $\frac{5}{9}$ m long.
Mrs Rose's scarf is $\frac{1}{5}$ m longer than Mr Rose's scarf.
How long is Mrs Rose's scarf?
How long are both the scarves altogether?

Mrs Rose's scarf is $\frac{34}{45}$ m long.

Both scarves together are $1\frac{14}{45}$ m long.

Fill in the boxes to make the calculation correct.

$$1\frac{\square}{10} = \frac{3}{\square} + \frac{\square}{10}$$

Various answers available. E.g.

$$1\frac{1}{10} = \frac{3}{5} + \frac{5}{10}$$



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# Add Fractions

## Notes and Guidance

Children explore adding mixed numbers. They look at different methods depending on whether the fractions total more than one. They add fractions with any denominators, building on their understanding from the previous steps.
Encourage children to draw bar models to support them in considering whether the fractions will cross the whole. They continue to simplify answers and convert between improper fractions and whole numbers when calculating.

## Mathematical Talk

How many wholes are there altogether?

Can you find the LCM of the denominators to find a common denominator?

Do you prefer Tommy or Whitney's method? Why?

Does Tommy's method work when the fractions add to more than one? How could we adapt his method?

Does Whitney's method work effectively when there are large whole numbers?

## Varied Fluency

Tommy is adding mixed numbers. He adds the wholes and then adds the fractions. Then, Tommy simplifies his answer.

$$1\frac{1}{2} + 2\frac{1}{6} = 1\frac{3}{6} + 2\frac{1}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$



Use Tommy's method to add the fractions.

$3\frac{1}{2} + 2\frac{3}{8} =$ $34\frac{1}{9} + 5\frac{2}{5} =$ $12\frac{5}{12} + 2\frac{1}{7} =$

Whitney is also adding mixed numbers. She converts them to improper fractions, adds them, and then converts them back to a mixed number.

$$1\frac{1}{2} + 2\frac{1}{6} = \frac{3}{2} + \frac{13}{6} = \frac{9}{6} + \frac{13}{6} = \frac{22}{6} = 3\frac{4}{6} = 3\frac{2}{3}$$



Use Whitney's method to add the fractions.

$3\frac{1}{2} + 2\frac{3}{8}$ $2\frac{1}{9} + 2\frac{2}{5}$ $2\frac{7}{9} + 2\frac{2}{5}$ $4\frac{3}{4} + 3\frac{11}{15}$

Jug A has $2\frac{3}{4}$ litres of juice in it. Jug B has $3\frac{4}{5}$ litres of juice in it.
How much juice is there in Jug A and Jug B altogether?

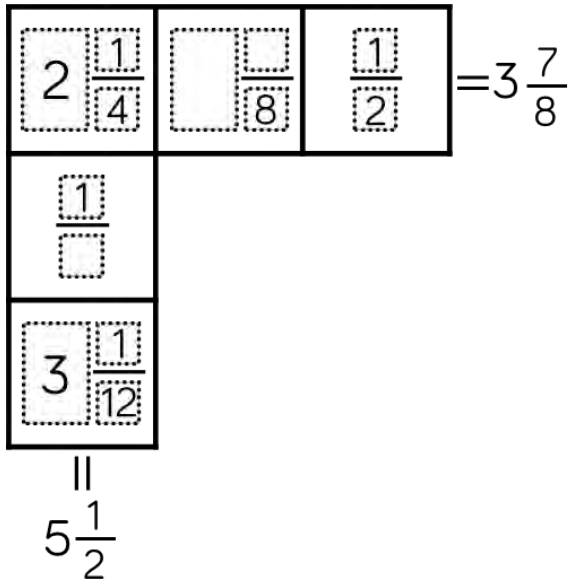


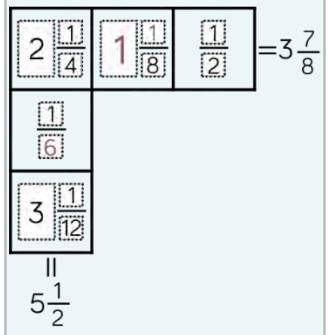
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# Add Fractions

## Reasoning and Problem Solving

Each row and column adds up to make the total at the end.
Use this information to complete the diagram.




Dora is baking muffins.
She uses $2\frac{1}{2}$ kg of flour, $1\frac{3}{5}$ kg of sugar and $1\frac{1}{4}$ kg of butter.

How much flour, sugar and butter does she use altogether?

How much more flour does she use than butter?

How much less butter does she use than sugar?

Dora uses $5\frac{7}{20}$ kg of flour, sugar and butter altogether.

Dora uses $1\frac{1}{4}$ kg more flour than butter.

Dora uses $\frac{7}{20}$ kg less butter than sugar.



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# Subtract Fractions

## Notes and Guidance

Children subtract mixed numbers. They explore different methods including exchanging wholes for fractions and subtracting the wholes and fractions separately and converting the mixed number to an improper fraction.
Encourage children to consider which method is the most efficient depending on the fractions they are subtracting. Bar models can support to help children to visualise the subtraction and understand the procedure.

## Mathematical Talk

How many eighths can we exchange for one whole?

What is the same about the first set of subtractions?

What is different about the subtractions? (How does this affect the subtraction?

Do you prefer Annie's or Amir's method? Why?

Look at Amir's calculation, what do you notice about the relationship between $3\frac{2}{5}$ and $1\frac{7}{10}$? ($3\frac{2}{5}$ is double $1\frac{7}{10}$)

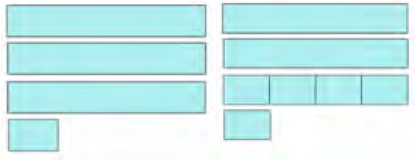
## Varied Fluency

Annie is calculating $3\frac{1}{4} - 1\frac{3}{4}$



I can't subtract the wholes and fractions separately because $\frac{1}{4}$ is less than $\frac{3}{4}$. I will exchange 1 whole for 4 quarters. $3\frac{1}{4} = 2\frac{5}{4}$

$3\frac{1}{4} - 1\frac{3}{4} = 2\frac{5}{4} - 1\frac{3}{4} = 1\frac{2}{4} = 1\frac{1}{2}$



Use Annie's method to calculate:

$3\frac{1}{8} - 1\frac{3}{8} =$ $3\frac{1}{8} - 1\frac{1}{2} =$ $3\frac{1}{8} - 1\frac{1}{5} =$ $3\frac{1}{8} - 1\frac{3}{5} =$

Amir is calculating $3\frac{2}{5} - 1\frac{7}{10}$

He converts the mixed numbers to improper fractions to subtract them.



$3\frac{2}{5} - 1\frac{7}{10} = \frac{17}{5} - \frac{17}{10} = \frac{34}{10} - \frac{17}{10} = \frac{17}{10} = 1\frac{7}{10}$

Convert the mixed numbers to improper fractions to calculate:

$4\frac{4}{5} - 1\frac{9}{10} =$ $2\frac{1}{7} - 1\frac{1}{3} =$ $3\frac{5}{12} - 1\frac{7}{9} =$ $3\frac{5}{11} - 1\frac{4}{5} =$

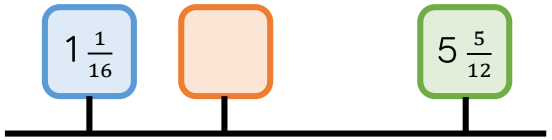


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# Subtract Fractions

## Reasoning and Problem Solving

A blue, orange and green box are on a number line.




The number in the green box is $3\frac{2}{3}$ more than the orange box.

The number in the orange box is:



The number in the orange box is  greater than the number in the blue box.

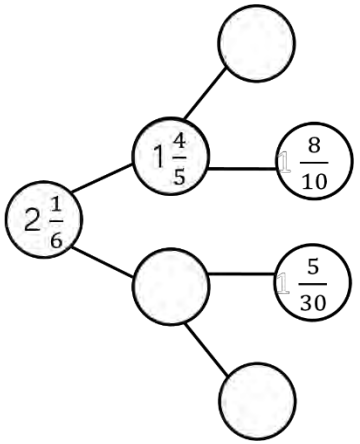
$5\frac{5}{12} - 3\frac{2}{3} = 1\frac{9}{12}$

The orange box is $1\frac{3}{4}$

$1\frac{3}{4} - 1\frac{1}{16} = \frac{11}{16}$

The orange box is $\frac{11}{16}$ greater than the blue box.

Complete the part-whole model.




$2\frac{1}{6} - 1\frac{4}{5} = 1\frac{11}{30}$

$1\frac{4}{5} - \frac{8}{10} = 1$

$1\frac{11}{30} - \frac{5}{30} = 1\frac{6}{30} = 1\frac{1}{5}$

Jack is calculating $4\frac{2}{7} - 2\frac{6}{7}$

He adds $\frac{1}{7}$ to both numbers.



$4\frac{2}{7} - 2\frac{6}{7} = 4\frac{3}{7} - 3$
so the answer is $1\frac{3}{7}$

Explain why Jack is correct.

Jack has increased both mixed numbers by $\frac{1}{7}$ so the difference has remained constant.



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# Mixed Addition & Subtraction

## Notes and Guidance

Children solve problems that involve adding and subtracting fractions and mixed numbers. Encourage children to consider the most efficient method of adding and subtracting fractions and to simplify their answers when possible.
Children can use bar models to represent the problems and support them in deciding whether they need to add or subtract. They can share their different methods to gain a flexible approach to calculating with fractions.

## Mathematical Talk

Can you draw a bar model to represent the problem? Do we need to add or subtract the fractions?

How do I know if my answer is simplified fully?

What is the lowest common multiple of the denominators?

How can I calculate the area covered by each vegetable? If you know the area for carrots and cabbages, how can you work out the area for potatoes? Can you think of 2 different ways?

## Varied Fluency

Alex has 5 bags of sweets.

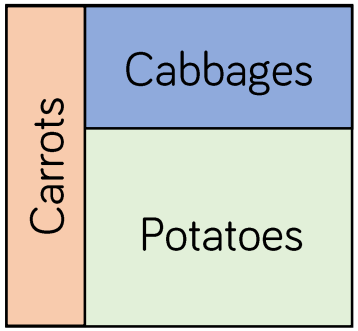
On Monday she eats $\frac{2}{3}$ of a bag and gives $\frac{4}{5}$ of a bag to her friend.

On Tuesday she eats $1\frac{1}{3}$ bags and gives $\frac{2}{5}$ of a bag to her friend.

What fraction of her sweets does Alex have left?

Give your answer in its simplest form.

Here is a vegetable patch. $\frac{1}{5}$ of the patch is for carrots. $\frac{3}{8}$ of the patch is for cabbages.




What fraction of the patch is for carrots and cabbages altogether?
What fraction of the patch is for potatoes?
What fraction more of the patch is for potatoes than cabbages?
Give your answers in their simplest form.

The vegetable patch has an area of 80 $m^2$
What is the area covered by each vegetable?



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# Mixed Addition & Subtraction

## Reasoning and Problem Solving

The mass of Annie's suitcase is $29\frac{1}{2}$ kg.

Teddy's suitcase is $2\frac{1}{5}$ kg lighter than Annie's.

How much does Teddy's suitcase weigh?

How much do the suitcases weigh altogether?

There is a weight allowance of 32 kg per suitcase.

How much below the weight allowance are Annie and Teddy?



Teddy's suitcase weighs $27\frac{3}{10}$ kg

The suitcases weigh $56\frac{4}{5}$ kg altogether.

Annie is $2\frac{1}{2}$ kg under the weight allowance.

Teddy is $4\frac{7}{10}$ kg under the weight allowance.

Find the value of the ☀

♥ $+ 3\frac{4}{9} = 6\frac{1}{3}$

$8\frac{1}{10} -$ ♥ $=$ ☀

The value of the ♥ is $2\frac{8}{9}$

The value of the ☀ is $5\frac{19}{90}$



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# Multiply Fractions by Integers

## Notes and Guidance

Children multiply fractions and mixed numbers by integers. They use diagrams to highlight the link between multiplication and repeated addition. This supports the children in understanding why the denominator stays the same and we multiply the numerator.

When multiplying mixed numbers, children partition into wholes and parts to multiply more efficiently. They compare this method with multiplying improper fractions.

## Mathematical Talk

How is multiplying fractions similar to adding fractions?

How does partitioning the mixed number into wholes and fractions support us to multiply?

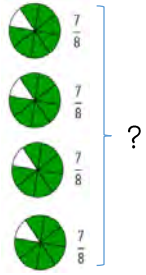
Do you prefer partitioning the mixed number or converting it to an improper fraction to multiply? Why?

Does it matter if the integer is first or second in the multiplication sentence? Why?

## Varied Fluency

Complete:

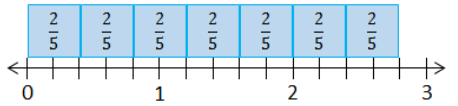
$4 \times \frac{7}{8}$ = ?



$3 \times \frac{2}{3}$ = ?



$\frac{2}{5} \times 7$



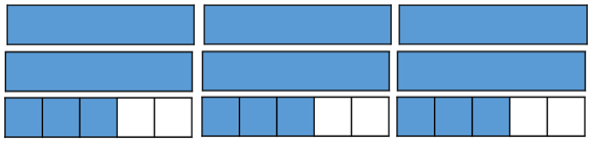

Eva partitions $2\frac{3}{5}$ to help her to calculate $2\frac{3}{5} \times 3$



$2 \times 3 = 6$

$\frac{3}{5} \times 3 = \frac{9}{5} = 1\frac{4}{5}$

$6 + 1\frac{4}{5} = 7\frac{4}{5}$



Use Eva's method to calculate:

$2\frac{5}{6} \times 3$ $\quad 1\frac{3}{7} \times 5$ $\quad 2\frac{2}{3} \times 3$ $\quad 4 \times 1\frac{1}{6}$

Convert the mixed number to an improper fraction to multiply.

$2\frac{3}{5} \times 3 = \frac{13}{5} \times 3 = \frac{39}{5} = 7\frac{4}{5}$

Use this method to calculate:

$3 \times 2\frac{2}{5}$ $\quad 1\frac{5}{7} \times 3$ $\quad 2 \times 1\frac{3}{4}$ $\quad 2 \times 1\frac{1}{6}$



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# Multiply Fractions by Integers

## Reasoning and Problem Solving

There are 9 lamp posts on a road. There is $4\frac{3}{8}$ of a metre between each lamp post.

What is the distance between the first and last lamp post?

$8 \times 4\frac{3}{8} = 8 \times \frac{35}{8}$

$= \frac{280}{8} = 35$

The distance between the first and last lamp post is 35 metres.

Use pattern blocks, if is equal to 1 whole, work out what fraction the other shapes represent.

Use this to calculate the multiplications.

Give your answers in their simplest form.

$\times 5 =$

$\times 5 =$

$\times 5 =$

$\times 5 = \frac{5}{6}$

$\times 5 = \frac{5}{3} = 1\frac{2}{3}$

$\times 5 = \frac{5}{2} = 2\frac{1}{2}$

Eva and Amir both work on a homework project.

Eva



I spent $4\frac{1}{4}$ hours a week for 4 weeks doing my project.

I spent $2\frac{3}{4}$ hours a week for 5 weeks doing my project.



Amir

Who spent the most time on their project?

Explain your reasoning.

$4 \times 4\frac{1}{4} = \frac{68}{4}$

$= 17$ hours

$5 \times 2\frac{3}{4} = \frac{55}{4}$

$= 13\frac{3}{4}$ hours

Eva spent $3\frac{1}{4}$ hours longer on her project than Amir did.



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# Multiply Fractions by Fractions

## Notes and Guidance

Children use concrete and pictorial representations to support them to multiply fractions. Support children in understanding the link between multiplying fractions and finding fractions of an amount: $\frac{1}{3} \times \frac{1}{2}$ is the same as $\frac{1}{3}$ of $\frac{1}{2}$

Encourage children to spot the patterns of what is happening in the multiplication, to support them in unpicking the procedure of multiplying fractions by multiplying the numerators and multiplying the denominators.

## Mathematical Talk

Could you use folding paper to calculate $\frac{2}{3} \times \frac{1}{2}$? How? Use a piece of paper to model this to a friend.

How are the diagrams similar to folding paper? Which do you find more efficient?

What do you notice about the product of the fractions you have multiplied? What is the procedure to multiply fractions?

Does multiplying two numbers always give you a larger product? Explain why.

## Varied Fluency

Dexter is calculating $\frac{1}{3} \times \frac{1}{2}$ by folding paper. He folds a piece of paper in half. He then folds the half into thirds.He shades the fraction of paper he has created. When he opens it up he finds he has shaded $\frac{1}{6}$ of the whole piece of paper.

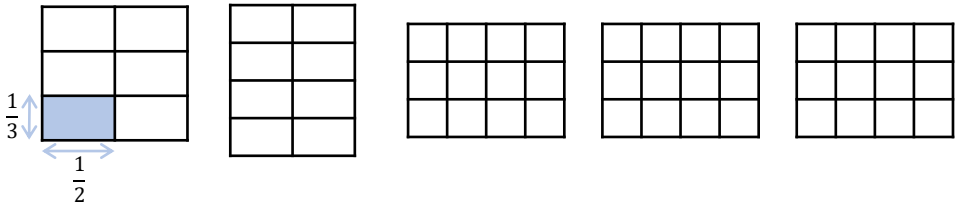


$\frac{1}{3} \times \frac{1}{2}$ means $\frac{1}{3}$ of a half. Folding half the paper into three equal parts showed me that $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$

Represent and calculate the multiplications by folding paper.

$\frac{1}{4} \times \frac{1}{2} =$ $\frac{1}{4} \times \frac{1}{3} =$ $\frac{1}{4} \times \frac{1}{4} =$

Alex is drawing diagrams to represent multiplying fractions.




Shade the diagrams to calculate:

$\frac{1}{3} \times \frac{1}{2} =$ $\frac{1}{4} \times \frac{1}{2} =$ $\frac{1}{3} \times \frac{1}{4} =$ $\frac{2}{3} \times \frac{1}{4} =$ $\frac{2}{3} \times \frac{3}{4} =$

Write your answers in their simplest form.

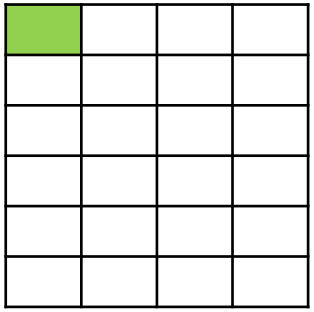


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# Multiply Fractions by Fractions

## Reasoning and Problem Solving

The shaded square in the grid below is the answer to a multiplying fractions question.
What was the question?



$\frac{1}{6} \times \frac{1}{4}$

How many ways can you complete the missing digits?

$\frac{\square}{\square} \times \frac{3}{\square} = \frac{6}{12}$

$\square \times \square = \frac{\square}{2}$

Possible answers:

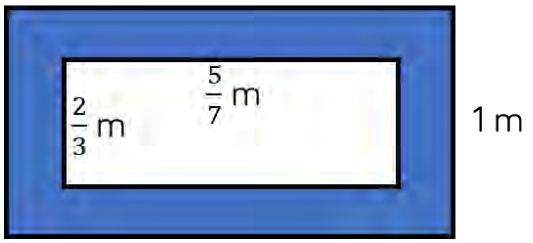
$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$

$\frac{2}{2} \times \frac{3}{6} = \frac{6}{12} = \frac{1}{2}$

Children could also use improper fractions.

Find the area of the shaded part of the shape.

Not drawn accurately




$1 \times 1 = 1$

$\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

$1 - \frac{10}{21} = \frac{11}{21}$

The shaded area is $\frac{11}{21}$ m$^2$.

Alex says,



$\frac{1}{4} \times \frac{1}{2}$ is the same as $\frac{1}{2}$ of a quarter.

Do you agree?
Explain why.

Alex is correct. Multiplication is commutative so $\frac{1}{4} \times \frac{1}{2}$ is the same as $\frac{1}{2}$ of a quarter or $\frac{1}{4}$ of a half.



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# Divide Fractions by Integers (1)

## Notes and Guidance

Children are introduced to dividing fractions by integers for the first time. They focus on dividing fractions where the numerator is a multiple of the integer they are dividing by. Encourage children to spot the pattern that the denominator stays the same and the numerator is divided by the integer
Children link dividing fractions to multiplying by unit fractions. Use the diagrams children drew for multiplying fractions to discuss how and why the calculations are similar.

## Mathematical Talk

How could you represent this fraction?
Is the numerator divisible by the integer?

Why doesn't the denominator change?

What pattern can you see when dividing elevenths?

How can we use the pattern to help us to calculate a mixed number by an integer? Can you convert it to an improper fraction?

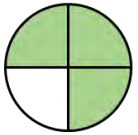
## Varied Fluency

Dexter has $\frac{2}{5}$ of a chocolate bar. He shares it with his friend. What fraction of the chocolate bar do they each get?

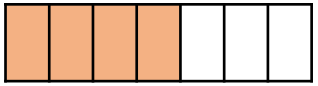


Use the diagrams to help you calculate.

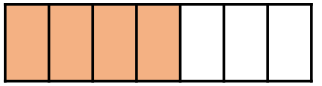
$\frac{3}{4} \div 3 =$



$\frac{4}{7} \div 4 =$



$\frac{4}{7} \div 2 =$



Calculate.

$\frac{1}{11} \div 1 =$ $\frac{2}{11} \div 2 =$ $\frac{3}{11} \div 3 =$ $\frac{4}{11} \div 4 =$

$\frac{2}{11} \div 2 =$ $\frac{4}{11} \div 2 =$ $\frac{6}{11} \div 2 =$ $\frac{8}{11} \div 2 =$

$\frac{3}{11} \div 3 =$ $\frac{6}{11} \div 3 =$ $\frac{9}{11} \div 3 =$ $1\frac{1}{11} \div 3 =$



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# Divide Fractions by Integers (1)

## Reasoning and Problem Solving

Tommy says,



Dividing by 2 is the same as finding half of a number so $\frac{4}{11} \div 2$ is the same as $\frac{1}{2} \times \frac{4}{11}$

Do you agree?
Explain why.

Tommy is correct. It may help children to understand this by reinforcing that $\frac{1}{2} \times \frac{4}{11}$ is the same as $\frac{1}{2}$ of $\frac{4}{11}$

Match the equivalent calculations.

| | |
|---|---|
| $\frac{1}{4} \times \frac{12}{13}$ | $\frac{12}{13} \div 2$ |
| $\frac{1}{6} \times \frac{12}{13}$ | $\frac{12}{13} \div 6$ |
| $\frac{1}{2} \times \frac{12}{13}$ | $\frac{12}{13} \div 4$ |
| $\frac{1}{3} \times \frac{12}{13}$ | $\frac{12}{13} \div 3$ |

$\frac{1}{4} \times \frac{12}{13} = \frac{12}{13} \div 4$

$\frac{1}{6} \times \frac{12}{13} = \frac{12}{13} \div 6$

$\frac{1}{2} \times \frac{12}{13} = \frac{12}{13} \div 2$

$\frac{1}{3} \times \frac{12}{13} = \frac{12}{13} \div 3$

Complete the missing integers.

$\frac{15}{16} \div \square = \frac{5}{16}$

$\frac{15}{16} \div \square = \frac{3}{16}$

$\frac{20}{23} \div \square = \frac{4}{23}$

$\frac{20}{23} \div \square = \frac{5}{23}$

3
5
5
4

Rosie walks for $\frac{3}{4}$ of an hour over 3 days.
She walks for the same amount of time each day.
How many minutes does Rosie walk each day?

Rosie walks for $\frac{1}{4}$ of an hour each day.
She walks for 15 minutes each day.



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# Divide Fractions by Integers (2)

## Notes and Guidance

Children divide fractions where the numerator is not a multiple of the integer they are dividing by.
They draw diagrams to divide fractions into equal parts and explore the link between multiplying by a unit fraction and dividing by an integer.
Children find equivalent fractions to support the divisions and draw diagrams to model how this works.

## Mathematical Talk

How is Mo's method of dividing fractions similar to multiplying $\frac{1}{3}$ by $\frac{1}{2}$?

Do you prefer Mo's or Annie's method? Explain why.

Why does finding an equivalent fraction help us to divide fractions by integers?

What multiplication can I use to calculate $\frac{3}{5} \div 2$? Explain how you know.

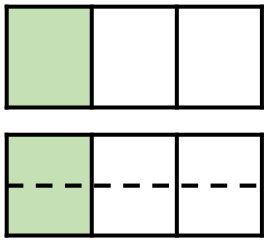
## Varied Fluency

Mo is dividing $\frac{1}{3}$ by 2



I have divided one third into 2 equal parts. Each part is worth $\frac{1}{6}$

$\frac{1}{3} \div 2 = \frac{1}{6}$



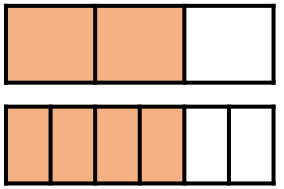
Draw diagrams to calculate:

$\frac{1}{3} \div 3 =$ $\frac{2}{3} \div 3 =$ $\frac{1}{5} \div 3 =$ $\frac{2}{5} \div 3 =$

Annie is dividing $\frac{2}{3}$ by 4



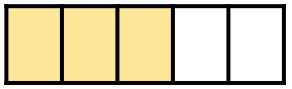
The numerator isn't a multiple of the integer I am dividing by so I will find an equivalent fraction to help me divide the numerator equally.



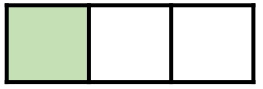
$\frac{2}{3} = \frac{4}{6}$ $\frac{4}{6} \div 4 = \frac{1}{6}$

Find equivalent fractions to calculate:

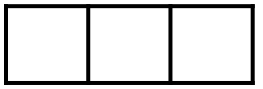
$\frac{3}{5} \div 2$



$\frac{1}{3} \div 3$



$\frac{2}{3} \div 3$





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# Divide Fractions by Integers (2)

## Reasoning and Problem Solving

Alex says,



I can only divide a fraction by an integer if the numerator is a multiple of the divisor.

Do you agree?
Explain why.

Alex is wrong, we can divide any fraction by an integer.

Calculate the missing fractions and integers.

$$\square \div 4 = \frac{7}{36}$$

$$\frac{3}{20} \div \square = \frac{3}{80}$$

$$\square \div \square = \frac{2}{5}$$

Is there more than one possibility?

$\frac{7}{9}$

4

There are many possibilities in this last question. Children could look for patterns between the fractions and integers.



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# Four Rules with Fractions

## Notes and Guidance

Children combine the four operations when calculating with fractions.
This is a good opportunity to recap the order of operations as children calculate equations with and without brackets.
Encourage children to draw bar models to represent worded problems in order to understand which operation they need to use?

## Mathematical Talk

Which part of the equation do we calculate first when we have more than one operation?

What do you notice about the six questions that begin with $3\frac{1}{3}$?

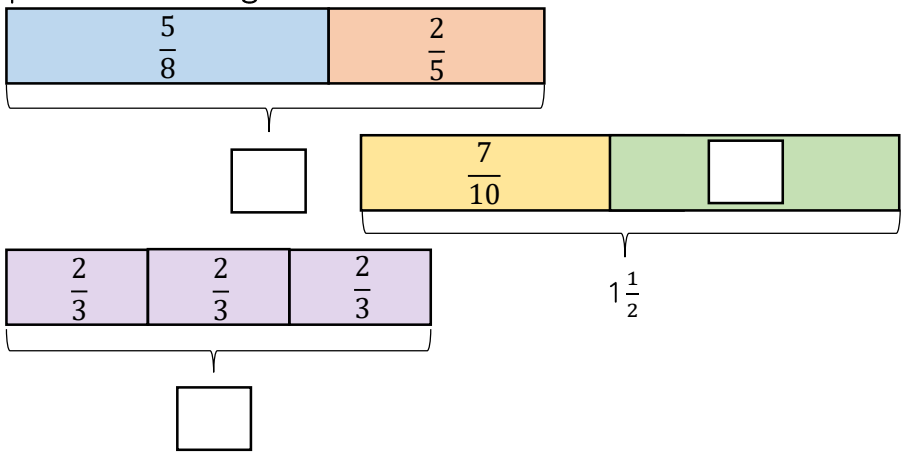
What's the same about the equations? What's different?

Which equation has the largest answer? Can you order the answers to the equations in descending order?

Can you write the worded problem as a number sentence?

## Varied Fluency

- Complete the missing boxes.




- Calculate:

$3\frac{1}{3}+\frac{1}{3}-2=$ $3\frac{1}{3}+\frac{1}{3}+2=$ $3\frac{1}{3}+\frac{1}{3}\times 2=$

$3\frac{1}{3}+\frac{1}{3}\div 2=$ $(3\frac{1}{3}+\frac{1}{3})\times 2=$ $(3\frac{1}{3}+\frac{1}{3})\div 2=$

- Jack has one quarter of a bag of sweets and Whitney has two thirds of a bag of sweets. They combined their sweets and shared them equally between themselves and Rosie.
What fraction of the sweets does each child receive?



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# Four Rules with Fractions

## Reasoning and Problem Solving

Add two sets of brackets to make the following calculation correct:

$$\frac{1}{2} + \frac{1}{4} \times 8 + \frac{1}{6} \div 3 = 6\frac{1}{18}$$

Explain where the brackets go and why.
Did you find any difficulties?

$(\frac{1}{2} + \frac{1}{4}) \times 8 + (\frac{1}{6} \div 3)$

Match each calculation to the correct answer.

| | |
|---|---|
| $(\frac{2}{3} + \frac{2}{9}) \div 4$ | $\frac{5}{9}$ |
| $\frac{2}{3} - \frac{1}{3} \div 3$ | $\frac{2}{9}$ |
| $\frac{1}{3} \times 2 - (1\frac{1}{9} \div 2)$ | $\frac{1}{9}$ |

$(\frac{2}{3} + \frac{2}{9}) \div 4 = \frac{2}{9}$

$\frac{2}{3} - \frac{1}{3} \div 3 = \frac{5}{9}$

$\frac{1}{3} \times 2 - (1\frac{1}{9} \div 2) = \frac{1}{9}$



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# Fraction of an Amount

## Notes and Guidance

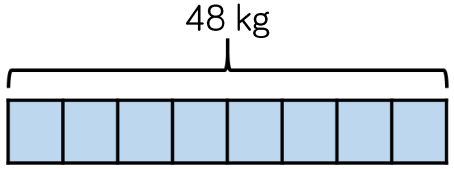
Children calculate fractions of an amount. They recognise that the denominator is the number of parts the amount is being divided into, and the numerator is the amount of those parts we need to know about.
Encourage children to draw bar models to support the procedure of dividing by the denominator and multiplying by the numerator to find fractions of amounts.

## Mathematical Talk

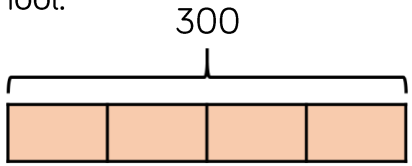
What is the value of the whole?
How many equal parts are there altogether?
How many equal parts do we need?
What is the value of each equal part?
Can you see a pattern in the questions starting with $\frac{1}{5}$ of 30?
What would the next column to the right of the questions be?
What would the next row of questions underneath be? How do you know? How can you predict the answers?

## Varied Fluency

A cook has 48 kg of potatoes. He uses $\frac{5}{8}$ of the potatoes. How many kilograms of the potatoes does he have left?
Use the bar model to find the answer to this question.




A football team has 300 tickets to give away.
They give $\frac{3}{4}$ of them to a local school.
How many tickets are left?




Calculate:

| | | | |
|---|---|---|---|
| $\frac{1}{5}$ of 30 = | $\frac{1}{5}$ of 60 = | $\frac{1}{5}$ of 120 = | $\frac{1}{5}$ of 240 = |
| $\frac{2}{5}$ of 30 = | $\frac{1}{5}$ of 600 = | $\frac{1}{10}$ of 120 = | $\frac{6}{5}$ of 240 = |
| $\frac{4}{5}$ of 30 = | $\frac{1}{5}$ of 6,000 = | $\frac{1}{20}$ of 120 = | $\frac{11}{5}$ of 240 = |

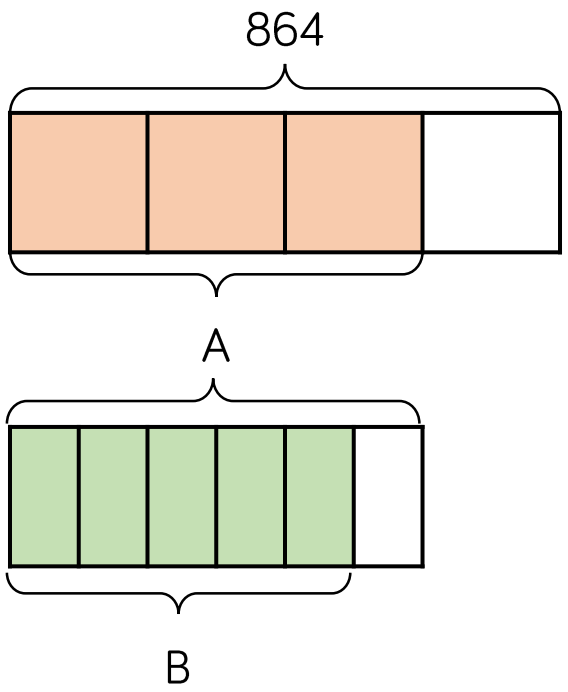


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# Fraction of an Amount

## Reasoning and Problem Solving

What is the value of A?
What is the value of B?




A = 648
B = 540

Two fashion designers receive $\frac{3}{8}$ of 208 metres of material.

One of them says:




Is she correct?
Explain your reasoning.

She is incorrect because 26 is only one eighth of 208 She needs to multiply her answer by 3 so that they each get 78 m each.

Calculate the missing digits.

$\frac{3}{8}$ of 40 = $\frac{?}{10}$ of 150

1

$\frac{1}{5}$ of 315 = $\frac{?}{8}$ of 72

7



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# Find the Whole

## Notes and Guidance

Children find the whole amount from the known value of a fraction. Encourage children to continue to use bar models to support them in representing the parts and the whole.
Children will consider looking for patterns when calculating the whole. Highlight the importance of multiplication and division when calculating fractions of amounts and how knowing our times-tables can support us to calculate the whole more efficiently.

## Mathematical Talk

How many equal parts are there altogether?

How many equal parts do we know?

What is the value of each equal part?

What is the value of the whole?

Can you see a pattern in the questions?

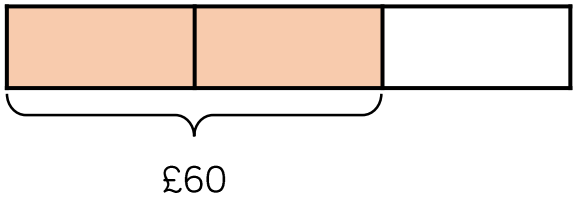
How can we find the whole?

Can you estimate what the answer is? Can you check the answer using a bar model?

## Varied Fluency

Jack has spent $\frac{2}{3}$ of his money.

He spent £60, how much did he have to start with?




Use a bar model to represent and solve the problems.

- Rosie eats $\frac{2}{5}$ of a packet of biscuits. She eats 10 biscuits. How many biscuits were in the original packet?
- In an election, $\frac{3}{8}$ of a town voted. If 120 people voted, how many people lived in the town?

Calculate:

$\frac{1}{4}$ of ___ = 12 $\frac{1}{4}$ of ___ = 36 $\frac{1}{4}$ of ___ = 108

$\frac{1}{12}$ of ___ = 12 $\frac{3}{4}$ of ___ = 36 $\frac{4}{4}$ of ___ = 108

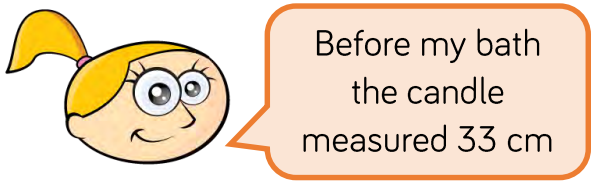


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# Find the Whole

## Reasoning and Problem Solving

Eva lit a candle while she had a bath.
After her bath, $\frac{2}{5}$ of the candle was left.
It measured 13 cm.
Eva says:




Is she correct?
Explain your reasoning.

She is incorrect.
$13 \div 2 = 6.5$
$6.5 \times 5 = 32.5$cm

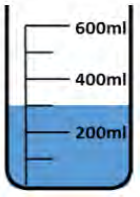
She either didn't halve correctly or didn't multiply correctly

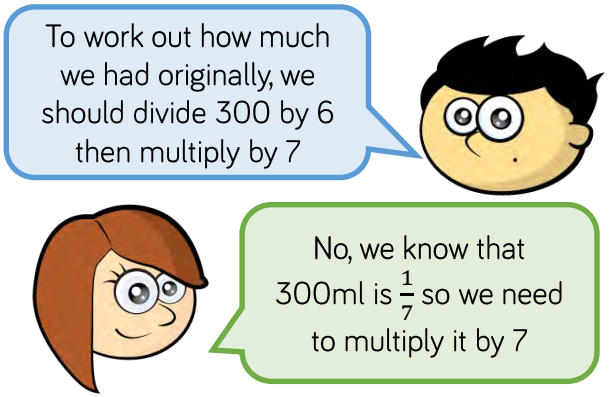
Write a problem which this bar model could represent.



Many possibilities.
$\frac{5}{8}$ of children have blue eyes. 15 children do not have blue eyes.
How many children are there altogether?

Rosie and Jack are making juice.
They use $\frac{6}{7}$ of the water in a jug and are left with this amount of water:




Who is correct?
Explain your reasoning.

Rosie is correct.
Jack would only be correct if $\frac{6}{7}$ was **remaining** but $\frac{6}{7}$ is what was used.
Rosie recognised that $\frac{1}{7}$ is left in the jug therefore multiplied it by 7 to correctly find the whole.



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Autumn - Block 4

# Position and Direction

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# Overview

## Small Steps

1. The first quadrant
2. Four quadrants
3. Translations
4. Reflections

## NC Objectives

Describe positions on the full coordinate grid (all four quadrants)

Draw and translate simple shapes on the coordinate plane, and reflect them in the axes.



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# The First Quadrant

## Notes and Guidance

Children recap work from Year 4 and Year 5 by reading and plotting coordinates in the first quadrant (the quadrant where both $x$ and $y$ coordinates are positive.).

Children draw shapes on a 2-D grid from given coordinates and may use their increasing understanding to write coordinates for shapes without plotting the points.

## Mathematical Talk

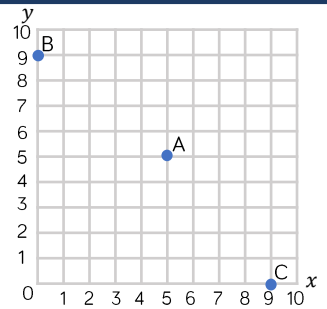
Which axis do we look at first?

Does joining up the vertices already given help you to draw the shape?

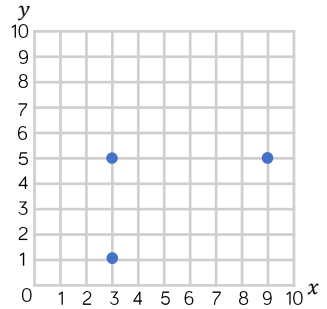
Can you draw a shape in the first quadrant and describe the coordinates of the vertices to a friend?

## Varied Fluency

- Whitney plots three coordinates.
  Write down the coordinates of points A, B and C.




- Tommy is drawing a rectangle on a grid.
  Plot the final vertex of the rectangle.
  Write the coordinate of the final vertex.




- Draw the vertices of the polygon with the coordinates (7, 1), (7, 4) and (10, 1)
  What type of polygon is the shape?

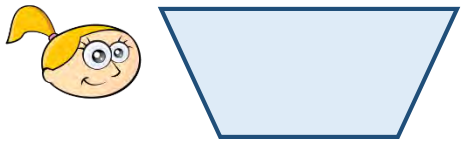


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# The First Quadrant

## Reasoning and Problem Solving

Eva is drawing a trapezium.
She wants her final shape to look like this:



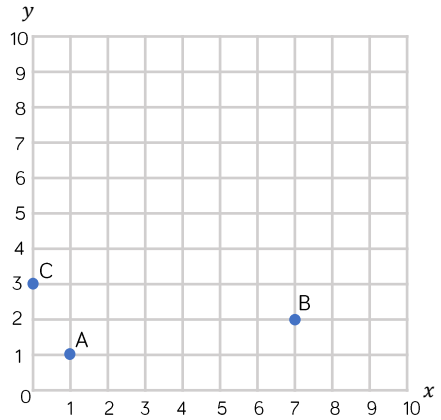
Eva uses the coordinates (2, 4), (4, 5), (1, 6) and (5, 6).
Will she draw the shape that she wants to?
If not, can you correct her coordinates?

Eva has plotted the coordinate (4, 5) incorrectly. This should be plotted at (4, 4) to make the trapezium that she wanted to draw (an isosceles trapezium).

Mo has written the coordinates of points A, B and C.

**A** (1, 1) **B** (2, 7) **C** (3, 0)

Mark Mo's work and correct his mistakes.




Explain why Mo could not make the same mistake for point A as he made for points B and C.

A is correct.

B and C have been plotted incorrectly because Mo has plotted the $x$ and $y$ coordinates the wrong way round.

Because the coordinates for point A are both the same number it does not matter if Mo incorrectly reads the $y$ coordinate as the first and the $x$ coordinate as the second.



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# Four Quadrants

## Notes and Guidance

Children extend their knowledge of the first quadrant to read and plot coordinates in all four quadrants.
They draw shapes from coordinates given.
Children need to become fluent in deciding which part of the axis is positive or negative.
Children need to develop understanding of how to find the length of a line by using the coordinates of its two endpoints.

## Mathematical Talk

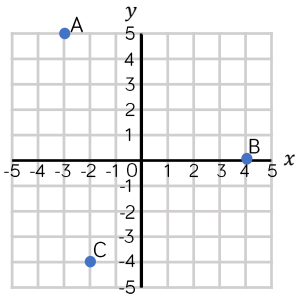
Which axis do we look at first?

If (0, 0) is the centre of the axis (the origin), which way do you move along the $x$-axis to find negative coordinates?

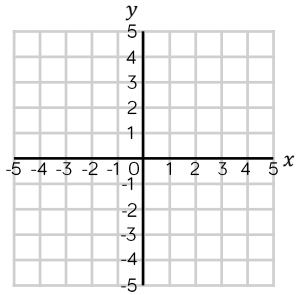
Which way do you move along the $y$-axis to find negative coordinates?

## Varied Fluency

Dora plotted three coordinates.
Write down the coordinates of points A, B and C.

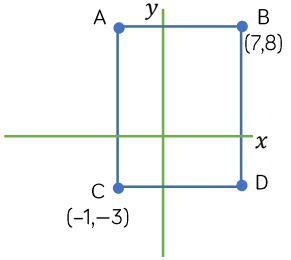



Draw a shape using the coordinates (-2, 2), (-4, 2), (-2, -3) and (-4, -2).
What is the name of shape?




Work out the missing coordinates of the rectangle.

What is the length of side AB?






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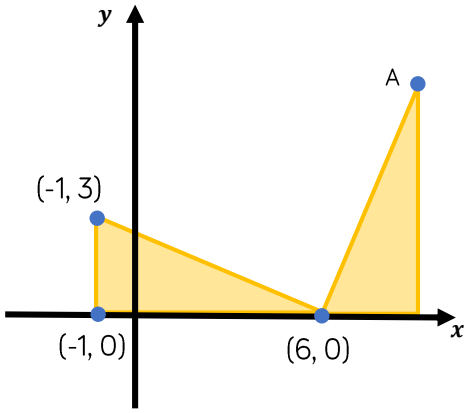
# Four Quadrants

## Reasoning and Problem Solving

The diagram shows two identical triangles.

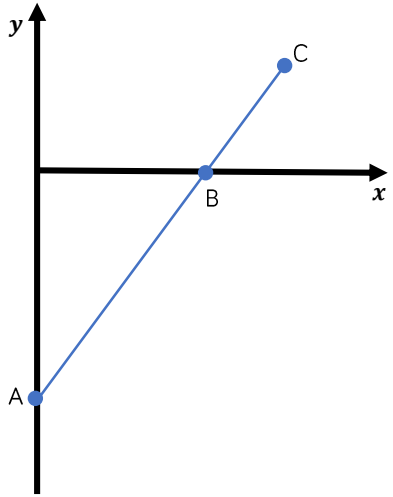
The coordinates of three points are shown.

Find the coordinates of point A.




(9, 7)

A is the point (0, – 10)
B is the point (8, 0)
The distance from A to B is two thirds of the distance from A to C.
Find the coordinates of C.




(12, 5)



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# Translations

## Notes and Guidance

Children use knowledge of coordinates and positional language to translate shapes in all four quadrants.

They describe translations using directional language, and use instructions to draw translated shapes.

## Mathematical Talk

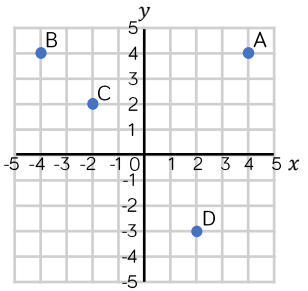
What does translation mean?

Which point are you going to look at when describing the translation?

Does each vertex translate in the same way?

## Varied Fluency

Use the graph to describe the translations.
One has been done for you.



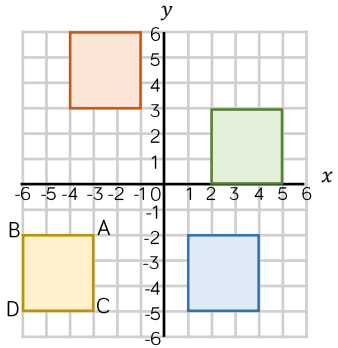

From **A** to **B** translate **8** units to the **left**.

From **C** to **D** translate __ units to the **right** and __ units **down**.

From **D** to **B** translate **6** units to the _____ and **7** units _____.

From **A** to **C** translate __ units to the ____ and __ units ____.

Write the coordinates for vertices A, B, C and D.
Describe the translation of ABCD to the blue square.




ABCD is moved 2 units to the right and 8 units up. Which colour square is it translated to?
Write the coordinates of the vertices of the translated shape.



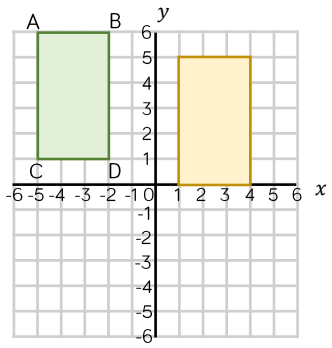
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# Translations

## Reasoning and Problem Solving

### True or False?

Dexter has translated the rectangle ABCD 6 units down and 1 unit to the right to get to the yellow rectangle.

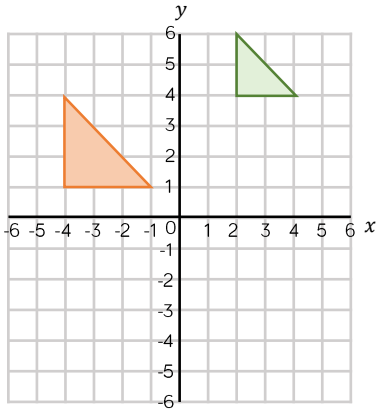



Explain your reasoning.

False.
The translation is 6 units to the right and 1 unit down.

### Spot the Mistake.

The green triangle has been translated 6 units to the left and 3 units down.




The triangle has changed size.
When a shape is translated its size does not change.

-6



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# Reflections

## Notes and Guidance

Children extend their knowledge of reflection by reflecting shapes in four quadrants. They will reflect in both the $x$-axis and the $y$-axis.

Children should use their knowledge of coordinates to ensure that shapes are correctly reflected.

## Mathematical Talk

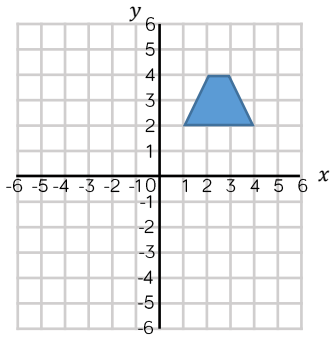
How is reflecting different to translating?

Can you reflect one vertex at a time? Does this make it easier to reflect the shape?

Which axis are you going to use as the mirror line?

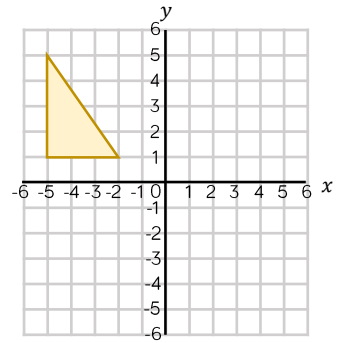
## Varied Fluency

Reflect the trapezium in the $x$-axis and then the $y$ –axis.
Complete the table with the new coordinates of the shape.




| | Reflected in the $x$-axis | Reflected in the $y$-axis |
|---|---|---|
| (1, 2) | | |
| (4, 2) | | |
| (2, 4) | | |
| (3, 4) | | |

Translate the shape 4 units to the right.
Then reflect the translated shape in the $y$-axis.



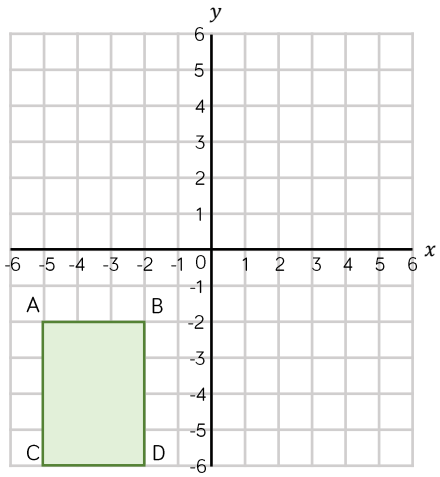



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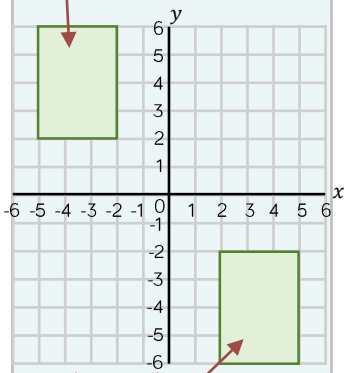
# Reflections

## Reasoning and Problem Solving

Rectangle ABCD is the result of a rectangle being reflected in either the $x$- or the $y$-axis.
Where could the original rectangle have been? Draw the possible original rectangles on the coordinate grid, and label the coordinates of each vertex.




The two original rectangles are:
Reflected in $x$-axis

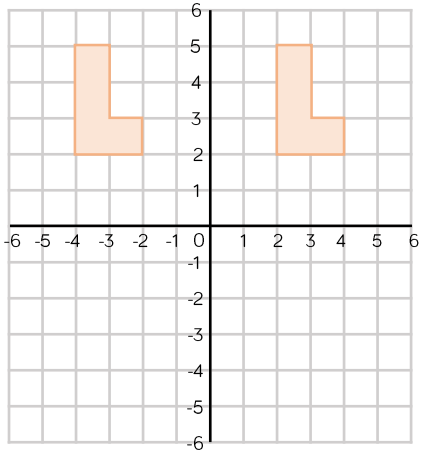



Reflected in $y$-axis

$x$-axis reflection original coordinates:
(-5, 6), (-2, 6), (-5, 2), (-2, 2)

$y$-axis reflection original coordinates:
(2, -2), (5, -2), (2, -6), (5, -6)

Annie has reflected the shape in the $y$-axis.
Is her drawing correct?
If not explain why.



Annie has used the correct axis, but her shape has not been reflected. She has just drawn the shape again on the other side of the axis.



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